9/26/2011. **Due:** Wednesday, Oct. 5, 2011 (in class).

Physics 501: Classical Mechanics

Read: **LL** 11-19; **JS** 4.1

Problems with stars are not for credit and will NOT be graded.

Homework 4

Exercise 1

Find the equation of an orbit of a particle of the mass m moving in the three-dimensional central field potential $U(r) = \frac{1}{2}m\omega^2 r^2$.

a) Write down the Lagrangian of the system using $\vec{r}$ to specify the position of the particle.

b) Use the conservation of the angular momentum to conclude that the orbits are planar.

c) Write down the conserved energy and the angular momentum of the problem using as coordinates r and ϕ - polar coordinates in the plane of the motion.

d) Using the results of b) and c) derive the equation of the orbit $r(\phi)$ in terms of quadratures. Calculate integrals and find $r(\phi)$ explicitly.

Exercise 2

How long would it take for the Moon to fall on the Earth if she suddenly lost its kinetic energy? Solve the problem without calculating any integrals.

Hint: Compare the circular orbit of the Moon with the (degenerate) elliptic orbit of the Moon falling on the Earth using the third Kepler's law.

Exercise 3

A comet is moving towards the Sun from far away with an impact parameter b and energy E. Find the perihelion of the orbit and the velocity of the comet at the perihelion.

Exercise 4

Find the differential scattering cross section of hard spheres of radii a scattered by an infinitely massive hard sphere of the radius R. Calculate the total cross section by integrating the differential one.

Exercise 5

Calculate the differential scattering cross section of particles of the energy E scattered by the central force potential $U(r) = -\frac{\alpha}{r} + \frac{\beta}{r^2}$ (α and β are positive constants).

Note: You can leave the result in the form of a transcendental equation for general scattering angles. Try to solve this equation and obtained the final result in the limit of small scattering angles.