Physics 501: Classical Mechanics

Read: **LL** 20-25;

Homework 5

Exercise 1

Beam and target particles interact through the repulsive potential $U(r) = \frac{k}{r^2}$. Find the differential cross section at small angles θ.

Exercise 2

Two blocks of masses m_1 and m_2 are connected by the spring which has potential energy $b(s^2 - a^2)^2$ and are free to move along the horizontal frictionless table. Here a, b are positive constants and s is the distance between blocks.

a) Write down the Lagrangian of this system in terms of $\vec{R}_1$, and $\vec{R}_2$ - the two-dimensional position-vectors of blocks.

b) Assume that the equilibrium positions of blocks are $(\pm a/2, 0)$ and introduce the deviations $\vec{r}_{1,2}$ from equilibrium positions. Write down the harmonic approximation of the Lagrangian of a) in terms of these deviations.

c) Find all eigenfrequences of the system.

Exercise 3

For the system of Ex. 2.

a) Find all normal modes of the system and express the Lagrangian of b) in terms of normal coordinates $Q_{1,2,3,4}$.

b) Explain the pecularities of the eigenspectrum found in Ex. 2 using the results of a) and symmetry considerations.

Exercise 4

N identical particles of mass m are free to move along the circle of radius R without any friction. Particles are connected by identical springs of spring constants k that lie along the arcs of the circle.

a) Write down the Lagrangian of the system using the angular deviations x_n $(n = 1, \ldots, N)$ of particle from the equilibrium N-gonal configuration.

b) Find all eigenfrequences of the system if $N = 2$.

c) Find all eigenfrequences of the system if $N = 3$.

d) Explain the presence of zero modes in b), c).

Exercise 5

For the system of Ex. 4.

a) Find all eigenfrequences of the system for general N.

b) Find normal coordinates and express the Lagrangian of the system in terms of normal coordinates for general N.

Hint: It is convenient to use $\tilde{x}_p = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} x_n e^{-i\frac{2\pi}{N}np}$ with $p = 0, 1, \ldots, N-1$ as coordinates instead of x_n.

c) Plot the eigenfrequency $\omega(q)$ versus $q = 2\pi p/N$. What happens in the limit $N \to \infty$?

Exercise 6 (JS 4.11)

a) For the undamped, one-freedom, linear oscillator with natural frequency ω_0, driven at frequency $\Omega \neq \omega_0$, find the rate at which energy is pumped into the oscillator. Find the maximum energy of an oscillator starting from rest at $q = 0$.

b) Do the same when the driving frequency is ω_0.