Homework 2.

Exercise 1: High temperature susceptibility

Consider the classical Hamiltonian:

$$\mathcal{H} = \sum_{i,j} J_{ij}\vec{S}_i\vec{S}_j + D\sum_i (S_i^z)^2, \tag{1}$$

where i, j are sites of three-dimensional stacked triangular lattice (simple Bravais lattice with lattice vectors $e_1 = (1,0,0)$, $e_2 = (1/2, \sqrt{3}/2, 0)$, $e_3 = (0,0,1)$. Non-zero exchange integrals are given by $J_{i,i\pm e_3} = J > 0$, $J_{i,i\pm e_{1,2}} = J_{i,i\pm(e_1-e_2)} = J' > 0$, and $D > 0$ is an easy plane anisotropy constant. Consider the following ratios between constants $J : D : J' = 500 : 5 : 1$.

a) In RPA approximation calculate the high temperature static susceptibility at wave vector q. Consider both cases: magnetic field is in (easy) plane and perpendicular to the plane.

b) What is the wave vector Q at which instability occurs when lowering the temperature? What is the critical temperature at which this transition occurs? Do you think the actual critical temperature is close to the one obtained in RPA? What is better estimate for critical temperature?

Hint. Remember that in purely one dimensional system ordering does not occur at all.

c) Find the uniform susceptibility at high temperatures. What happens with this susceptibility at the critical temperature found in b)?

Exercise 2: Classical ground state

For the spin Hamiltonian given in exercise 1 find the classical ground state (spin configuration minimizing the Hamiltonian). Find the uniform susceptibility $\chi_{||}$ (magnetic field is applied parallel to z-axis) at zero temperature.

Hint. Use the value Q obtained in exercise 1.

Exercise 2′: Not for grading

Find $\chi_\perp$ (magnetic field is in plane) at zero temperature for the system from exercises 1,2. Find $\chi_\perp$ (magnetic field is in plane) at zero temperature and at finite magnetic field $J'S \ll h \ll JS$.

Exercise 3: Spin waves

For the same system find the spectrum of spin waves at very low temperatures (assuming that spins are classical). How many branches of spin waves have you obtained? How many Goldstone modes have you found? Find (or estimate) the thermal corrections to the average spin on lattice site (calculate $\langle \vec{S}_i \rangle$ at very low temperature. Which spin wave modes contribute the most to these thermal corrections?