Physics 682.01:
Topics in Condensed Matter Physics
Quantum magnetism

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Table of values

Value and Units	**Item**
$k = 1.4 \times 10^{-16} erg/K$	Boltzmann constant
$\hbar = 1.05 \times 10^{-27} erg \cdot s$	Planck constant
$c = 3.0 \times 10^{10} cm/s$	velocity of light
$e = 4.8 \times 10^{-10} esu$	electron charge
$m = 9.1 \times 10^{-28} g$	electron mass
$\mu_B = 0.93 \times 10^{-20} erg/gauss$	Bohr magneton $\mu_B = \frac{e\hbar}{2mc}$
$a_B = 0.53\text{Å} = 5.3 \times 10^{-9} cm$	Bohr radius $a_B = \frac{\hbar^2}{me^2}$

Part I

What is Quantum Magnetism?

1 There is no classical magnetism

Before studying “quantum magnetism” let us try to derive magnetism from purely classical physics

1.1 Bohr-van Leeuwen Theorem

Consider the system of N classical (charged) particles. A general Hamiltonian for such a system in magnetic field

$$\mathcal{H} = \sum_{k=1}^{N} \frac{1}{2m_k} \left(\vec{p}_k - \frac{e_k}{c} \vec{A}_k \right)^2 + V(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N). \tag{1}$$

Here $\vec{r}_k, \vec{p}_k$ are positions and momenta of particles and $\vec{A}_k = \vec{A}(\vec{r}_k)$ is a vector potential at the position of k-th particle. Particles move in external magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$. E.g., one can choose vector potential in symmetric gauge $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$. The total magnetization $\hat{\vec{M}}$ of the system is

$$\hat{\vec{M}} = -\frac{\partial \mathcal{H}}{\partial \vec{B}}, \tag{2}$$

where “hat” $\hat{}$ in classical expression means that the quantity is a function of coordinates and momenta. We find total average magnetization.

$$\vec{M} = \langle \hat{\vec{M}} \rangle = \frac{1}{Z} \int \prod_{k=1}^{N} d\vec{p}_k d\vec{r}_k \; \hat{\vec{M}} e^{-\beta \mathcal{H}} = \frac{1}{\beta} \frac{\partial}{\partial \vec{B}} \ln Z, \tag{3}$$

where $\beta = \frac{1}{kT}$ and Z is a partition function

$$Z = \int \prod_{k=1}^{N} d\vec{p}_k d\vec{r}_k \; e^{-\beta \mathcal{H}}. \tag{4}$$

We write partition function as

$$\begin{aligned} Z &= \int \prod_{k=1}^{N} d\vec{p}_k d\vec{r}_k \; e^{-\beta \left(\sum_{k=1}^{N} \frac{1}{2m_k} \left(\vec{p}_k - \frac{e_k}{c} \vec{A}_k \right)^2 + V(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \right)} \\ &= \int \prod_{k=1}^{N} d\vec{p}_k d\vec{r}_k \; e^{-\beta \left(\sum_{k=1}^{N} \frac{\vec{p}_k^2}{2m_k} + V(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \right)} = Z(\vec{B} = 0). \end{aligned} \tag{5}$$

To obtain the last line we shifted $\vec{p}_k$ by $\frac{e}{c}\vec{A}_k$ under the integral.

We see from (5) that partition function Z does not depend on external magnetic field $\vec{B}$. Therefore, from (3) we obtain

$$\vec{M} = \frac{1}{\beta}\frac{\partial}{\partial \vec{B}} \ln Z = 0, \tag{6}$$

because $\ln Z$ does not depend on $\vec{B}$.

We obtained that

Purely classical system does not have a magnetization even in the presence of magnetic field.

The statement of the absence of magnetism for purely classical system is often called **Bohr-van Leeuwen theorem**. This theorem tells that magnetism is always quantum.

1.2 Dipole interaction between quantum magnetic moments

We show that the dipole interaction between atomic magnetic moments is too weak to explain strong magnetism, e.g., ferromagnetism. Namely, if we assume that nearest atoms have magnetic moment of the order of Bohr magneton and are at the distance $2-3$Å, then the dipole interaction is of the order of $0.1-1$K. This is too weak interaction to explain ordering temperatures of the order of 1000K.

On the other hand Coulomb interaction at distance $2-3$Å is very strong. This can serve as a hint that the origin of strong magnetic moment interaction is in Coulomb interaction.

2 Exchange interaction

2.1 Direct exchange. Ferromagnetic and antiferromagnetic interactions

2.2 Superexchange

2.3 Double exchange

3 Spin Hamiltonians

In this section we follow chapter 2.3 of Ref.[4].

3.1 Lande tensor, single-ion anisotropy, and Van Vleck paramagnetism

On the example of transition metal ions. Zero order Hamiltonian is intraatomic Coulomb interaction and crystal field. Zeeman coupling and spin-orbit coupling are treated in second order perturbation theory.

3.2 Exchange anisotropy

4 Conclusion

Spin Hamiltonians considered have been derived from quantum theory. However, after spin Hamiltonian is written down it can be treated (semi)classically. In what follows we will review such classical treatments. However, we call "quantum magnetism" magnetic physics which appear as a consequence of discrete (quantum) nature of spin. Therefore, in the main part of the course we consider magnetic phenomena in which quantum fluctuations are strong.

Part II

Classical Magnetism

5 Curie-Weiss law

5.1 Single spin in magnetic field

5.2 Weiss molecular field approximation (mean field)

6 Static susceptibility. RPA approximation

Definition of static susceptibility. Relation with spin-spin correlators.

In this section we follow Ref.[4], Chapter 4.

6.1 High temperatures

Derivation of static susceptibility in RPA approximation.

6.1.1 Ferromagnetism

6.1.2 Antiferromagnetism

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9.1.1 Kinks in 1d

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9.5 Topological classification of defects and textures

9.6 Topological vs. non-topological phase transition. Classical Heisenberg model in 3d

Part III

Quantum Magnetism

10 Equivalence between classical systems in $d+1$ dimensions and quantum systems in d dimensions

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12.2 Path integral

12.2.1 Canonic pair

12.2.2 Heisenberg algebra

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14.3 Non-linear sigma model with theta term

14.4 Spin chain with $S = 1$

14.5 AKLT Hamiltonians

14.6 Quasi-one-dimensional magnets

15 Spin ladders

16 Two-dimensional quantum spin systems

References

[1] Assa Auerbach, *Interacting Electrons and Quantum Magnetism*, Springer-Verlag, New York, 1994.
QC176.8.E4 A94 1994

[2] Eduardo Fradkin, *Field Theories of Condensed Matter Systems*, Perseus Books, Reading, Massachusetts, 1998.
QC173.4.C65 F54 1990

[3] A.O. Gogolin, A.A. Nersesyan, A.M. Tsvelik, *Bosonization and Strongly Correlated Systems*, Cambridge University Press, 1998.
QC793.5.B62 G64 1998

[4] Robert M. White, *Quantum Theory of Magnetism*, McGraw-Hill, Inc, 1970.
QC754.2.Q34 W48 1983