Physics 540: Statistical Mechanics I

Read: LL 56-59
Problems to study: K.4 pr 7

"**LL** 1" means section 1 from Landau and Lifshitz book
"**K**.1 pr 2" means problem 2 from section 1 of Kubo's book.

Homework 10

Exercise 1

Given the concentration $n = N/Area$ of a two-dimensional ideal Fermi gas (with spin $s = 1/2$) find its Fermi wavevector $k_F = p_F/\hbar$ and Fermi energy ϵ_F. What is the density of states $\nu(\epsilon_F)$ of such a gas at Fermi energy.

The Quantum Hall effect is observed at low temperatures in a quasi-two-dimensional electron gas, usually created in doped semiconductors. A typical two-dimensional electron density in such system is $2 \times 10^{11}\mathrm{cm}^{-2}$. Calculate numerically for this "free electron" gas k_F, ϵ_F (in eV), and degeneracy temperature T_F (in K). Assume that the effective electron band mass is just a free electron mass.

Exercise 2

Calculate the specific heat of a two-dimensional ideal electron gas at very low temperatures. Express the result in terms of the electron density of states at Fermi energy.

Exercise 3

a) Calculate the spin paramagnetic (Pauli) susceptibility of a two-dimensional ideal electron gas at very low temperatures (strong degeneracy). Express the result in terms of the electron density of states at Fermi energy.
b) The same as a) but at high temperatures (weak degeneracy).
c) Explain the physical meaning of the qualitative difference between values of susceptibility obtained in a) and b). Explain also the meaning of the ratio of these values.

Exercise 4

Consider an ideal two-dimensional electron gas of concentration n in perpendicular magnetic field. Calculate exactly the energy of the ground state of such a gas as a function of magnetic field $E(n, T = 0, B)$. Calculate and plot M/B versus $1/B$, where M is the magnetization (per particle) of the gas. What is the period of de Haas-van Alphen oscillations.