2/26/02. **Due:** Tuesday, March 5, 2002 (in class).

Physics 540: Statistical Mechanics I

Read: LL 28,31,35,36
Problems to study: K.1 ex 10, **K**.2 problems 1,9

"**LL** 1" means section 1 from Landau and Lifshitz book

Homework 4

Exercise 1

Macroscopic system is defined by its quantum Hamiltonian $\hat{H}(V,N,B) = \hat{H}_0(V,N) - B\hat{\mathcal{M}}(V,N)$ which depends on "external parameters" such as volume V, number of particles N, and external magnetic field B. The operator $\hat{\mathcal{M}}$ is a "magnetization operator". We define partition function

$$Z(T,V,N,B) = \mathrm{Tr}\left(e^{-\beta\hat{H}}\right),$$

free energy

$$F(T,V,N,B) = -T\ln Z,$$

magnetization

$$\mathcal{M} = \left\langle \hat{\mathcal{M}} \right\rangle_{T,V,N},$$

and magnetic susceptibility

$$\chi = \left(\frac{\partial \mathcal{M}}{\partial B}\right)_{T,V,N},$$

where $\beta = 1/T$ is an inverse temperature.

a) Express the susceptibility in terms of free energy of the system using only the above definitions. Write down dF in terms of canonic variables T,V,N,B and their conjugates.

b) Find the fluctuations of magnetization $\langle(\delta\mathcal{M})^2\rangle \equiv \langle(\hat{\mathcal{M}} - \mathcal{M})^2\rangle$ in terms of magnetic susceptibility. Derive the thermodynamic inequality $\chi > 0$.

c) Write down the expression (as the trace of some operator) for the partition function $Z(T,V,N,\mathcal{M})$. What is the differential $dY(T,V,N,\mathcal{M})$ of the thermodynamic potential Y as $Y = -T\ln Z(T,V,N,\mathcal{M})$ in terms of its canonic variables and their conjugates?

Exercise 2

For a system of N spin-1/2 particles in a magnetic field (see **K**.1 ex 10)

$$Z(T, N, B) = [2\cosh(\mu B/T)]^N .$$

Calculate the magnetic susceptibility of the system and find the fluctuations (variance) of magnetization $(\delta\mathcal{M})^2$ as a function of temperature and magnetic field. Find this variance in limits corresponding to high and low temperatures.