Name: ______________________________

Recitation section (circle one)

R01 Mon. 12:50 **R02 Wed. 12:50** **R03 Mon. 3:50**

Final Exam

Closed book. No notes allowed. **Any** calculators are permitted. There are no trick questions here. There are five questions, one per page, each worth **20 points**, for a maximum score of **100**. In general, you need to show the reasoning behind your answer. A correct number alone will not get much credit.

Do the exam on these pages. It would be smart to write your name on all pages, in case of staple failure. If a question seems too easy, perhaps it is just easy. Relax and good luck! Two significant figures will be accepted as sufficient on this test. Do not forget to give units in your final answer. If your calculator fails, you should get **numerical** answers to 10% by simple arithmetics for almost full credit. **The exams will be graded by Tuesday, May 16 and can be looked at in Sasha Abanov's office (B-102).**

Turn cell phones off. Have your University ID out for checking, please.

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total Score	

Total score for the course (out of 100)

Grade

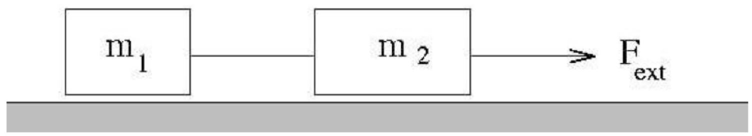
Problem 1

Two masses $m_1 = 10.0$ kg and $m_2 = 14.0$ kg, are coupled by a massless cord. They slide on a horizontal surface. An external force $F_{ext} = 25.0$ N is applied to mass m_2 as shown.

a) Suppose there is no friction. Find the acceleration.

b) Find the tension in the cord.

c) Now suppose that there is friction, determined by a coefficient μ_k of kinetic friction which is the same for each mass. The system slides to the right at constant speed $v = 1.2$ m/s. What is the coefficient of friction μ_k?



Solution

a) **(6 points)** Newton's laws for the masses

$$\begin{aligned} T &= m_1 a, \\ F_{ext} - T &= m_2 a. \end{aligned}$$

Adding these gives $F_{ext} = (m_1 + m_2)a$ and solving for a

$$a = \frac{F_{ext}}{m_1 + m_2} = \frac{25.0}{10.0 + 14.0} \approx 1.04 \text{ m/s}^2.$$

b) **(6 points)**

$$T = m_1 a = 10.0 \cdot 1.04 \approx 10.4 \text{ N}.$$

c) **(8 points)** System of two masses moves without acceleration. The net external force should be zero.

$$F_{ext} - \mu_k (m_1 + m_2) g = 0.$$

$$\mu_k = \frac{F_{ext}}{(m_1 + m_2)g} = \frac{25.0}{(10.0 + 14.0)9.8} \approx 0.11.$$

Problem 2

A discus thrower can throw a discus (on the Earth, of course) over 70 m distance (close to the world record). Will the athlete be able to throw a discus with an escape speed on the surface of an asteroid of density 2500 kg/m^3 and diameter of 200 km? Assume that the space suite is very light and does not hinder athlete's movements. To solve this problem:

a) Find the initial speed of the discus from the given range.

b) Find the escape velocity from the asteroid.

c) Compare those velocities and come to a conclusion.

Hint: the range of the projectile's motion is given by $R = \frac{v_0^2 \sin 2\alpha_0}{g}$. The constant $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

Solution

a) **(6 points)** The range of the projectile motion is given by $R = \frac{v_0^2 \sin 2\alpha_0}{g}$ and is maximal for $\alpha_0 = 45^o$. The maximal range is $R_{max} = \frac{v_0^2}{g}$. We obtain for the initial velocity of the discus

$$v_0 = \sqrt{gR_{max}} = \sqrt{70 \cdot 9.8} \approx 26.2 \text{ m/s}.$$

b) **(12 points)** First of all, let us calculate the mass of the asteroid. Its radius $r = d/2 = 100$ km $= 1.0 \times 10^5$ m. Its volume is given approximately by $\frac{4}{3}\pi r^3$. The mass of the asteroid is

$$M_A = \rho \frac{4}{3}\pi r^3 = 2500\frac{4}{3}3.14(1.0 \times 10^5)^3 \approx 1.05 \times 10^{19} \text{ kg}.$$

The escape velocity can be found from

$$-G\frac{M_A m}{r} + \frac{m v_{esc}^2}{2} = 0,$$

where M_A is the mass of the asteroid and r is its radius. We have

$$v_{esc} = \sqrt{\frac{2GM_A}{r}} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 1.05 \cdot 10^{19}}{1.0 \cdot 10^5}} \approx 120 \text{ m/s}.$$

c) **(2 points)** Comparing velocities we see that $v_0 < v_{esc}$, and the athlete will not be able to send the discuss into the outer space from the asteroid.

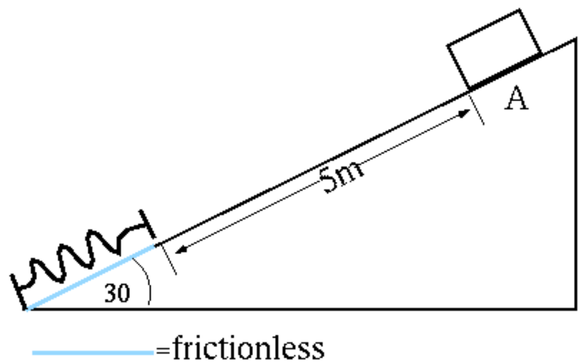
Problem 3

A mass $m = 5.0$kg is released from the point A and it slides down the incline ($\theta = 30^o$), where coefficient of kinetic friction is 0.3. It goes distance $L = 5.0$ m and hits an ideal spring with a spring constant $k = 3000$N/m. While it is being acted upon by the spring, assume it is on a frictionless surface.

a) What is the maximal compression x of the spring?

b) How far does the block go up the plane on the rebound from the spring?

Hint: In calculations you can assume that the maximal compression of the spring is small $x \ll L$.



Solution

We are going to use the energy conservation principle to solve this problem. Let us choose the zero of potential energy at the initial position of the block.

a) **(10 points)** If 1 is the initial position and 2 is the position where the compression of the string is maximal, we have

$$K_1 + U_{grav1} + U_{spr1} + W_{fr} = K_2 + U_{grav2} + U_{spr2}.$$

For initial position we have

$$K_1 = U_{grav1} = U_{spr1} = 0.$$

If the maximal compression of the spring x we have at the point of maximal compression $y = -(L + x)\sin\theta$ ($L = 5$ m) and

$$\begin{aligned} K_2 &= 0, \\ U_{grav2} &= -mg(L + x)\sin\theta, \\ U_{spr2} &= kx^2/2. \end{aligned}$$

The magnitude of the friction force is $F_{fr} = \mu_k mg\cos\theta$ and the work done by friction is

$$W_{fr} = -L\mu_k mg\cos\theta.$$

Plugging all this into the first equation we have

$$0 + 0 + 0 - L\mu_k mg\cos\theta = 0 - mg(L + x)\sin\theta + \frac{kx^2}{2}.$$

Let us assume that x is small $x \ll L$ and neglect x in the left hand side of this equation (alternatively you can solve this quadratic equation and obtain exact but very close result). We solve

$$x = \sqrt{\frac{2mg}{k} L(\sin\theta - \mu_k \cos\theta)} = \sqrt{\frac{2\ 5.0\ 9.8}{3000} 5.0(\sin 30^o - 0.3\cos 30^o)} \approx 0.2 \text{ m}.$$

Indeed, this compression is much smaller than 5m and our assumption has been justified.

b) **(10 points)** If 3 is the position of the block when it stops after rebound we have

$$K_1 + U_{grav1} + U_{spr1} + W_{fr} = K_3 + U_{grav3} + U_{spr3}.$$

If it goes distance s up the slope we have

$$\begin{aligned} K_3 &= 0, \\ U_{grav3} &= -mg(L-s)\sin\theta, \\ U_{spr2} &= 0. \end{aligned}$$

and the work done by friction

$$W_{fr} = -(L+s)\mu_k mg\cos\theta.$$

We have

$$0 + 0 + 0 - (L+s)\mu_k mg\cos\theta = 0 - mg(L-s)\sin\theta + 0.$$

and

$$s = L\frac{\sin\theta - \mu_k\cos\theta}{\sin\theta + \mu_k\cos\theta} = 5\frac{\sin 30^o - 0.3\cos 30^o}{\sin 30^o + 0.3\cos 30^o} \approx 1.6 \text{ m}.$$

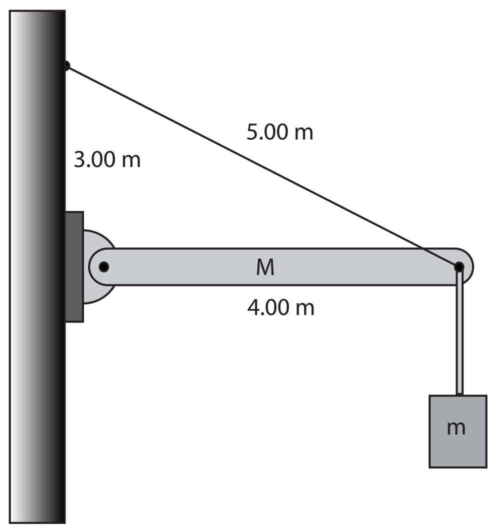
Problem 4

The horizontal beam shown in the figure has a mass of $M = 20$kg, and its center of gravity is at its center. The mass of the load $m = 10$kg. The load is connected to the beam by a very light rod. Find

a) the tension T in the 5.00 m cable,

b) the horizontal and vertical components of the force exerted on the beam at the wall,

c) the change of the length of the 5.00 m cable due to tension. The cross-sectional area of the cable is $A = 1.0$ mm^2 and it is made of copper (Y=11×10^{11} Pa).



Solution

Consider beam-load as a system. We are interested only in external forces acting on this system. These are: weights of the beam Mg, weight of the load mg, tension of the 5.00 m cable T, the horizontal component of the force from the wall N (directed to the right), the vertical component U of the same force.

We choose the direction downwards for the axis y and the direction to the right for the axis x as positive. Also we choose the clockwise sense of rotation as positive. We calculate all torques about the pivot point.

We write three equilibrium conditions

$$\text{(3 points)} \quad \sum F_x = 0: \quad N - T\cos\theta = 0,$$
$$\text{(3 points)} \quad \sum F_y = 0: \quad Mg + mg - U - T\sin\theta = 0,$$
$$\text{(3 points)} \quad \sum \tau_z = 0: \quad Mg\frac{L}{2} + mgL - T\sin\theta L = 0.$$

Here L is the length of the beam, and θ is the angle between the cable and the beam. We find from the figure $\sin\theta = 3/5$, $\cos\theta = 4/5$.

a) The last equation gives us immediately

$$\text{(3 points)} \qquad T = \frac{(m + M/2)g}{\sin\theta} = \frac{(10 + 20/2)9.8}{3/5} \approx 3.3 \times 10^2 \text{ N}.$$

b) Using the found value of T and solving first two equations we find

$$
\begin{aligned}
&\text{(2 points)} \qquad U = \quad Mg + mg - T\sin\theta = \frac{1}{2}Mg = \frac{1}{2}209.8 = 98 \text{ N}, \\
&\text{(2 points)} \qquad N = \quad T\cos\theta = 3.3\times 10^2\frac{4}{5} = 2.6\times 10^2 \text{ N}. \qquad (1)
\end{aligned}
$$

c) (4 points)

The relation between tensile stress and strain is

$$\frac{T}{A} = Y\frac{\Delta l}{l_0},$$

where $l_0 = 5.00$ m is the length of an unstretched cable and $A = 1.0 \text{ mm}^2 = 1.0\times 10^{-6} \text{ m}^2$ is its cross-sectional area. We find

$$\Delta l = l_0\frac{T/A}{Y} = 5.00\frac{3.3\times 10^2/10^{-6}}{11\times 10^{11}} = 1.5\times 10^{-3} \text{ m} = 1.5 \text{ mm}.$$

Problem 5

Consider again the system described in the previous problem. Assume that the 5.00 m cable breaks and the beam pivots around the pin with no friction.

a) What is its initial angular acceleration α of the beam, just after the cable breaks?

b) What is its initial linear acceleration a of the load, just after the cable breaks?

c) At the same moment, what is the tension of the very light rod F_T connecting the load m with the beam?

Assume that the rod stays vertical as it is freely connected to the beam (on a hinge).

Hint: The moment of inertia of a uniform bar about an axis through its center is $I = \frac{1}{12}ML^2$ and about an axis through one end is $I = \frac{1}{3}ML^2$.

Solution

We write dynamic equations of motion (second Newton's law) for beam and the load separately.

For the beam

$$\text{(5 points)} \qquad \sum \tau_z = I\alpha : \qquad Mg\frac{L}{2} + F_T L = I\alpha,$$

where F_T is the tension of the rod connecting the load and the beam.

For the load

$$\text{(5 points)} \qquad \sum F_y = ma : \qquad mg - F_T = ma.$$

(2 points) In addition we have an expression for the moment of inertia of the beam about the pivot point $I = \frac{1}{3}ML^2$ and the kinematic relation between angular acceleration of the beam and the linear acceleration of the load $a = L\alpha$. Plugging this relations in above two equations we obtain

$$\begin{aligned} Mg\frac{L}{2} + F_T L &= \frac{1}{3}ML^2\alpha, \\ mg - F_T &= mL\alpha, \end{aligned}$$

a) (3 points)

We cancel L in the first equation

$$\frac{1}{2}Mg + F_T = \frac{1}{3}ML\alpha$$

and add it to the second equation. We see that the tension force cancels and we obtain

$$\frac{1}{2}Mg + mg = \frac{1}{3}ML\alpha + mL\alpha$$

and

$$\alpha = \frac{m + M/2}{m + M/3}\frac{g}{L} = \frac{10 + 20/2}{10 + 20/3}\frac{9.8}{4} \approx 2.9\ \frac{\text{rad}}{\text{s}^2}.$$

b) (2 points)

$$a = \alpha L \approx 11.8\ \frac{\text{m}}{\text{s}^2}.$$

We notice that this acceleration is bigger than g!

c) (3 points)

The equation for the load gives us

$$F_T = mg - mL\alpha = m(g - a) = 10(9.8 - 11.8) \approx -20 \text{ N}.$$

We notice, that it is negative! Indeed, for the load to move with the acceleration larger than g the rod should push it down!