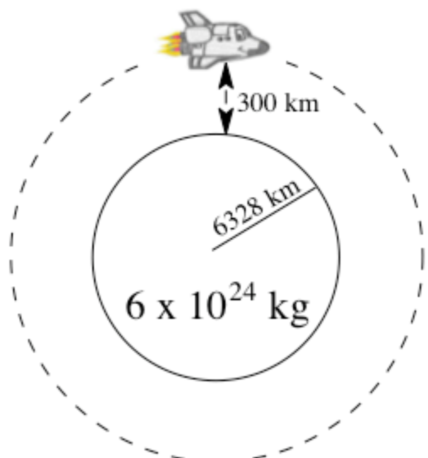
Physics 125: Classical Physics A

1 Practice Problems for Final Exam

Problem 1

The Space Shuttle flies with a typical altitude of 300 km above the surface of the earth. The radius of earth is 6328 km and the mass of earth is 6×10^{24} kg.



Not to Scale :)

(a) What is the velocity of the shuttle?

(b) What is the period of the shuttle's orbit?

Solution

a)

What is the velocity of the shuttle?

ANSWER: OK, this is a single force circular motion problem:

$$G\frac{mM}{R^2} = m\frac{v^2}{R}$$
$$v = \sqrt{\frac{GM}{R}}$$
$$R = 6328km + 300km = 6628km = 6628000m$$
$$v = \sqrt{\frac{6.67 \times 10^{-11} 6 \times 10^{24}}{6628000}}$$
$$v = 7770\frac{m}{s}$$

b)

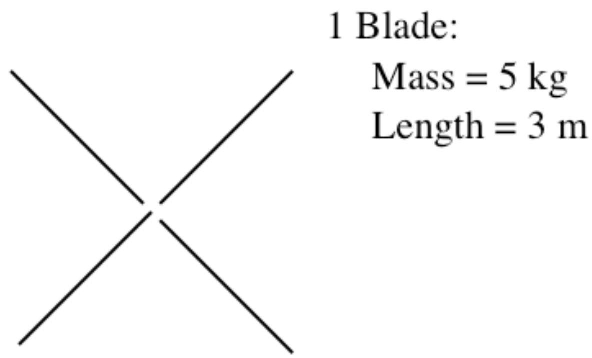
What is the period of the shuttle's orbit?

ANSWER: We simply relate the velocity to the period:

$$v = \frac{2\pi R}{\tau}$$
$$\tau = \frac{2\pi R}{v}$$
$$\tau = \frac{2\pi 6628000}{7770}$$
$$\tau = 5360 sec$$

Problem 2

Shown in the figure below is a four blade propeller. Each blade has a mass of 5 kg and a length of 3 meters. The propeller starts from rest and reaches 200 RPM after 20 seconds.



4–Blade Propeller

(a) What is the angular acceleration of the propeller in $\frac{rad}{sec}$?

(b) How many turns has the propeller undergone in these 20 seconds?

(c) The speed of sound in air is 345 $\frac{m}{s}$. At what ω does the tip of the propeller exceed the speed of sound?

(d) Assuming the angular acceleration is constant at what time does the tip of the propeller exceed the speed of sound?

(e) Approximate the propeller as four rods rotating about their ends. What is the moment if inertia, I, of the propeller?

(f) What torque is required to make the propeller move as detailed in this problem?

Solution

(a) What is the angular acceleration of the propeller in $\frac{rad}{sec}$?
ANSWER: OK. First we will repair the units on the angular velocity:

$$\omega = 200rpm \times \frac{2\pi rad}{1rev} \times \frac{1min}{60sec} = 20.94\frac{rad}{s} \tag{57}$$

$$\tag{58}$$

Now we can write the information in a well-formed manner:

$$t(\omega = 20.94) = 20 \tag{59}$$

$$\omega = \omega_o + \alpha t \tag{60}$$

$$20.94 = 0 + \alpha 20 \tag{61}$$

$$\alpha = \frac{20.94}{20} = 1.047\frac{rad}{s^2} \tag{62}$$

(b) How many turns has the propeller undergone in these 20 seconds?
ANSWER: OK, turns is a question about θ:

$$\theta(t = 20) = ? \tag{63}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{64}$$

$$\theta = 0 + 0 + \frac{1}{2}1.047(20^2) \tag{65}$$

$$\theta = 209rad \times \frac{1turn}{2\pi rad} = 33.33turns \tag{66}$$

(c) The speed of sound in air is 345 $\frac{m}{s}$. At what ω does the tip of the propeller exceed the speed of sound?

ANSWER: We can relate the linear velocity of the top to the angular velocity by:

$$v = r\omega \tag{67}$$

$$\omega_{sound} = \frac{v}{r} = \frac{345}{3} = 115\frac{rad}{sec} \tag{68}$$

(d) Assuming the angular acceleration is constant at what time does the tip of the propeller exceed the speed of sound?

ANSWER: OK, this is asking for t under a condition on ω:

$$t(\omega = 115) = ? \tag{69}$$

$$\omega = \omega_0 + \alpha t \tag{70}$$

$$115 = 0 + 1.047t \tag{71}$$

$$t = \frac{115}{1.047} = 109.8sec \tag{72}$$

(e) Approximate the propeller as four rods rotating about their ends. What is the moment if inertia, I, of the propeller?

ANSWER: Each rod has a moment of inertia of $\frac{1}{3}ML^2$. There are four rods total:

$$I_1 = \frac{1}{3}ML^2 = \frac{1}{3}5(3)^2 = 5 \times 3 = 15kgm^2 \tag{73}$$

$$I_4 = 4 \times I_1 = 60kgm^2 \tag{74}$$

(f) What torque is required to make the propeller move as detailed in this problem?

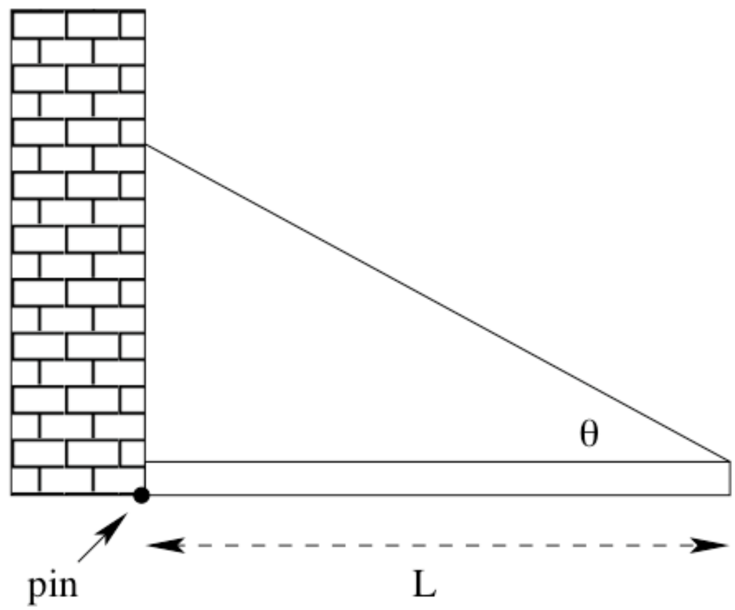
ANSWER:

$$\tau = I\alpha \tag{75}$$

$$\tau = 60 \times 1.047 = 62.82Nm \tag{76}$$

Problem 3

A uniform bar of mass M = 2.5 kg and length L = 2m is supported in static equilibrium in a horizontal position by a pin at its left end attached to the wall, and a string attached at the right end which makes and angle relative to the bar of $\theta = 35^o$.



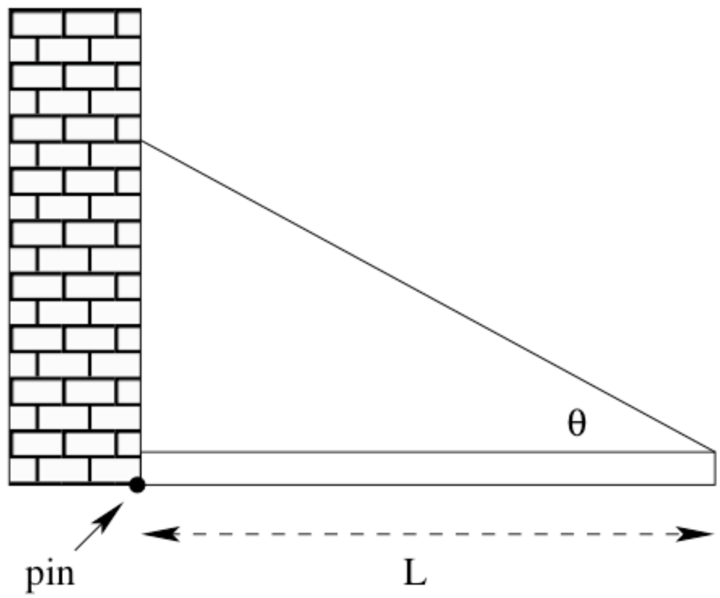
(a) Find the tension in the string.

(b) The string breaks and the bar pivots around the pin with no friction. What is its initial angular acceleration, α, just after the string breaks? (The moment of inertia of a uniform bar about an axis through its center is $I = \frac{1}{12}ML^2$ and about an axis through one end is $I = \frac{1}{3}ML^2$).

Solution

(a) Find the tension in the string.

OK...this is a force AND torque problem (static equilibrium). We must draw and extended free body diagram:



FINE. Now we have to do THREE equations: F_x, F_y, τ. We'll start in the x direction:

$$\sum F_x = ma_x = 0 \tag{77}$$

$$N - T\cos\theta = 0 \tag{78}$$

Now for the y direction:

$$\sum F_y = ma_y = 0 \tag{79}$$

$$U + T\sin\theta - mg = 0 \tag{80}$$

Finally, we need to do the torques. For the torques, we need to decide which axis the object is not rotating around. ALL OF THEM. We'll take the left end of the rod as the axis and then solve the problem:

$$\sum \tau = I\alpha = 0 \tag{81}$$

$$mg\frac{L}{2} - T\sin\theta L = 0 \tag{82}$$

$$2.5(9.8)\frac{2}{2} = T\sin 35\ 2 \tag{83}$$

$$T = \frac{2.5(9.8)}{2\sin 35} = 21.4N \tag{84}$$

We could then use this in the previous equations to find **U** and **N**.

The string breaks and the bar pivots around the pin with no friction. What is its initial angular acceleration, α, just after the string breaks? (The moment of inertia of a uniform bar about an axis through its center is $I = \frac{1}{12}ML^2$ and about an axis through one end is $I = \frac{1}{3}ML^2$).

OH!!! This is so easy! Once the string breaks...only the mg force makes a torque!!

$$\sum \tau = I\alpha \tag{85}$$

$$mg\frac{L}{2} = I\alpha \tag{86}$$

$$mg\frac{L}{2} = \frac{1}{12}mL^2\alpha \tag{87}$$

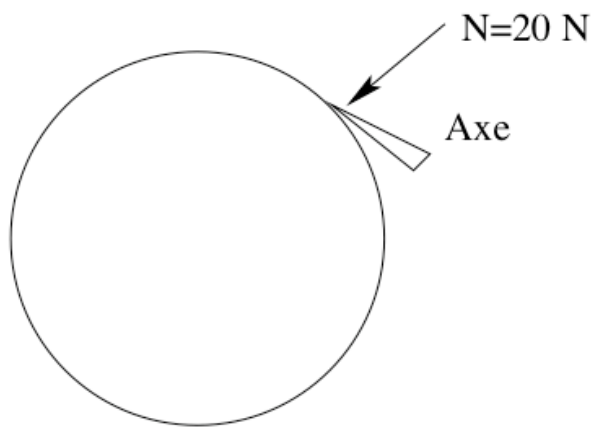
m cancels as well as one of the L's:

$$g\frac{1}{2} = \frac{1}{3}L\alpha \tag{88}$$

$$\alpha = \frac{3g}{2L} = 7.35\frac{rad}{s^2} \tag{89}$$

Problem 4

Shown in the figure below is a grindstone (mass = 80 kg, radius = 0.7 m) that is used to sharpen an axe. The axe is pressed against the spinning grindstone with a normal force of 20 N. When this is done the grindstone slows from an initial motion of 100 rpm and comes to a stop in 2 minutes.



Stone Wheel

(a) What is the angular acceleration of the grindstone?

(b) How many turns does the grindstone make before coming to rest?

(c) What is the moment of inertia of the grindstone?

(d) What is the coefficient of kinetic friction between the axe and the grindstone?

Solution

(a) What is the angular acceleration of the grindstone? **ANSWER:** Again a kinematics problem that is done by writing the information carefully:

$$t(\omega = 0) = 120sec \tag{90}$$

$$\omega_0 = 100rpm\frac{2\pi rad}{1rev}\frac{1min}{60sec} = 10.47\frac{rad}{s} \tag{91}$$

$$\omega = \omega_0 + \alpha t \tag{92}$$

$$0 = 10.47 + \alpha 120 \tag{93}$$

$$\alpha = -\frac{10.47}{120} = -0.087\frac{rad}{s^2} \tag{94}$$

(b) How many turns does the grindstone make before coming to rest? **ANSWER:** OK, this asks about theta:

$$\theta(\omega = 0) = ? \tag{95}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \tag{96}$$

$$0 = 10.47^2 + 2(-0.087)\theta \tag{97}$$

$$\theta = \frac{10.47^2}{2 \times 0.087} = 630rad \tag{98}$$

$$\theta = 630rad\frac{1turn}{2\pi rad} = 100turns \tag{99}$$

(c) What is the moment of inertia of the grindstone? **ANSWER:**

$$I = \frac{1}{2}MR^2 = \frac{1}{2}80(0.7^2) = 19.6kgm^2 \tag{100}$$

(d) What is the coefficient of kinetic friction between the axe and the grindstone? **ANSWER:**

$$\tau = I\alpha \tag{101}$$

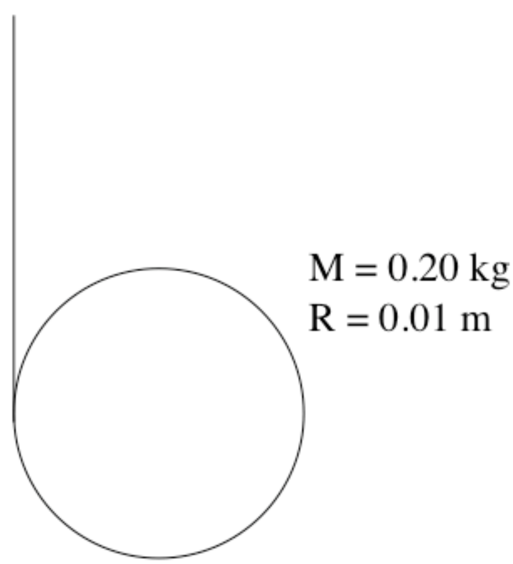
$$\tau = 19.6 \times 0.087 = 1.7052 \tag{102}$$

$$\tau = FR = \mu_k NR = 1.7052 \tag{103}$$

$$\mu_k = \frac{1.7052}{20 \times 0.7} = 0.12 \tag{104}$$

Problem 5

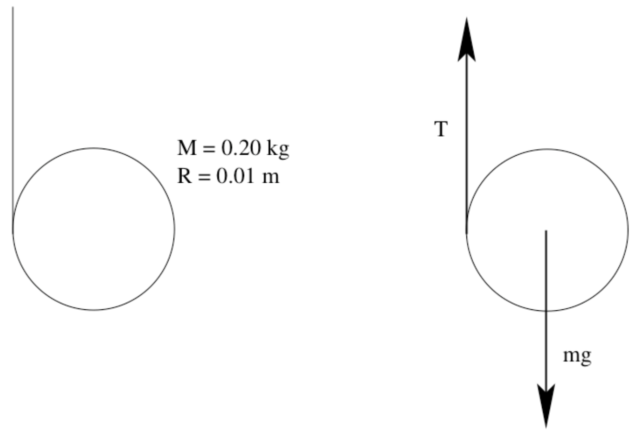
Shown in the figure below is a simple yoyo. The string is wrapped around the outside of the yoyo and the yoyo accelerates downward.



(a) What is the acceleration of the yoyo?

(b) What is the tension in the string?

Solution



This problem has one linear and one rotational dimension:

$$y: \tag{105}$$
$$mg - T = ma \tag{106}$$

$$\tau: \tag{107}$$
$$rT = I\alpha \tag{108}$$
$$I = \frac{1}{2}Mr^2 \tag{109}$$
$$\alpha = \frac{a}{r} \tag{110}$$
$$rT = \frac{1}{2}Mr^2\frac{a}{r} \tag{111}$$
$$T = \frac{1}{2}ma \tag{112}$$

Adding these results:

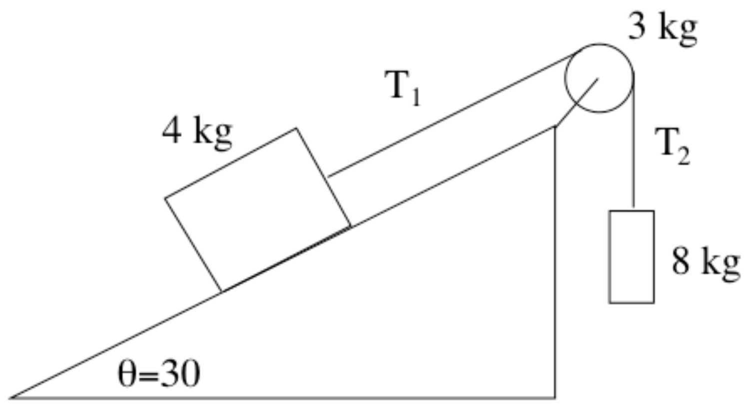
$$mg = 1.5ma \tag{113}$$
$$a = \frac{g}{1.5} = 6.533\frac{m}{s^2} \tag{114}$$

Putting this in the second equation:

$$T = \frac{1}{2}ma = \frac{1}{2}0.2 \times 6.533 = 0.6533N \tag{115}$$

Problem 6

Shown in the figure below is a system of masses and pulleys. NOTE: The pulley has mass and so the tensions on the two sides of the pulley are not the same! Please approximate the pulley as a solid cylinder.

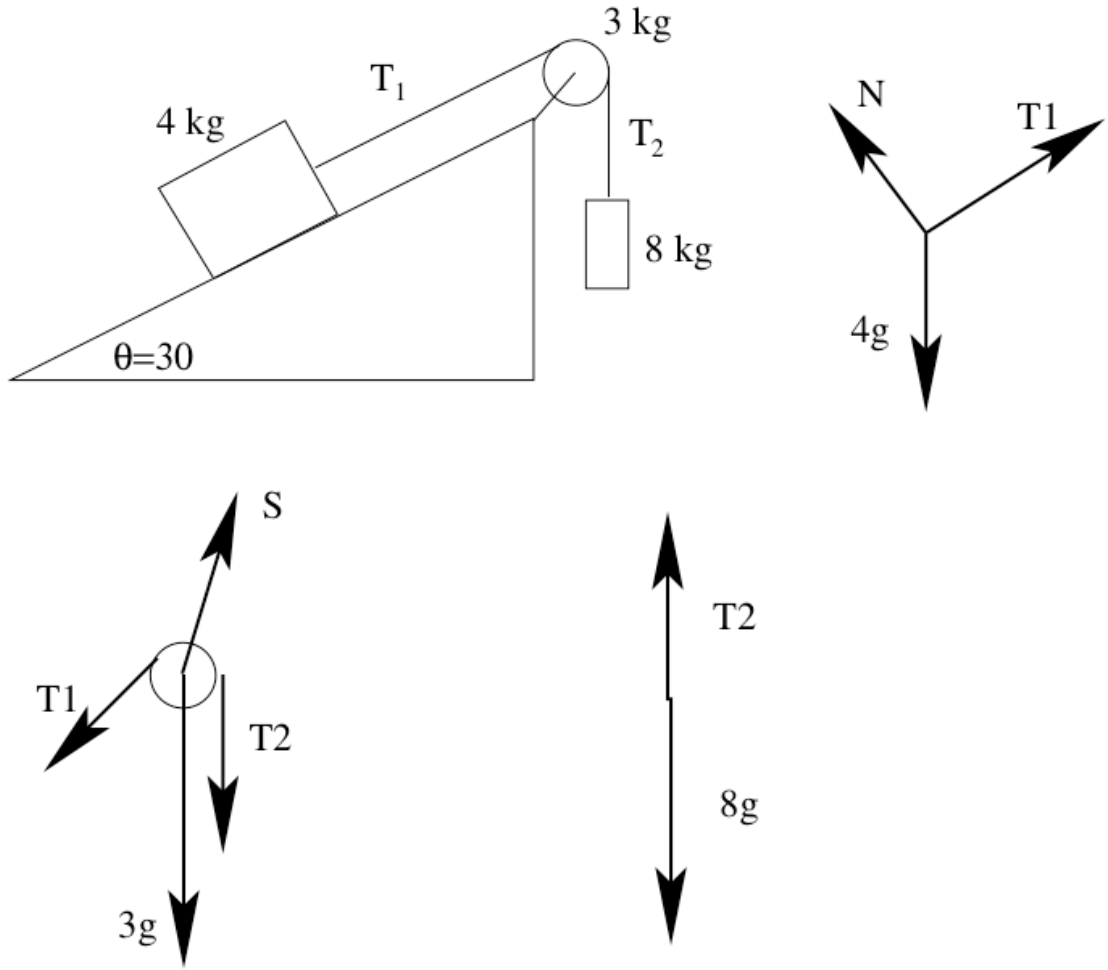


(a) What is the linear acceleration of the system?

(b) What is the tension T_1?

(c) What is the tension T_2?

Solution



OK, this problem has three objects and therefore 3 free body diagrams as shown above. First we'll deal with the 4 kg mass:

$$y: \tag{116}$$

$$N - 4g\cos\theta = 0 \tag{117}$$

$$N = 4g\cos theta = 33.95N \tag{118}$$

$$x: \tag{119}$$

$$T_1 - 4g\sin\theta = 4a \tag{120}$$

$$\tag{121}$$

For 8 kg mass we will have:

$$y: \tag{122}$$

$$8g - T_2 = 8a \tag{123}$$

Finally for the pulley, we could not care less about the force of the shaft, S, and so we'll only ONLy do the rotational direction:

$$\tau: \tag{124}$$

$$rT_2 - rT_1 = I\alpha \tag{125}$$

$$I = \frac{1}{2}mr^2 \tag{126}$$

$$\alpha = \frac{a}{r} \tag{127}$$

$$r(T_2 - T_1) = \frac{1}{2}mr^2\frac{a}{r} \tag{128}$$

$$T_2 - T_1 = \frac{1}{2}ma = 1.5a \tag{129}$$

OK, adding them all up we find that the tensions cancel!

$$8g - 4g\sin\theta = 13.5a \tag{130}$$

$$6g = 13.5a \tag{131}$$

$$a = \frac{6g}{13.5} = 4.355\frac{m}{s^2} \tag{132}$$

Plugging into the first equation:

$$T_1 - 4g\sin\theta = 4a \tag{133}$$

$$T_1 - 2g = 4 \times 4.355 \tag{134}$$

$$T_1 = 37.02N \tag{135}$$

Plugging into the second equation:

$$8g - T_2 = 8a \tag{136}$$

$$8g - T_2 = 8 \times 4.355 \tag{137}$$

$$T_2 = 8g - 8 \times 4.355 = 43.56N \tag{138}$$

Problem 7

10. A race between two cylinders is performed. Both cylinders begine at the top of the 1.5 m tall ramp and roll without slipping to the bottom.

(a) What is the speed of the hollow cylinder when it reaches the flat?

(b) What is the speed of the solid cylinder when it reaches the flat?

(c) Which one would win the race?

Solution

(a) **What is the speed of the hollow cylinder when it reaches the flat?**
ANSWER: This is a case of conservation of energy:

$$TE_1 = TE_2 \tag{139}$$

$$0 + 0 + mgy + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0 + 0 \tag{140}$$

$$I = mr^2 \tag{141}$$

$$\omega = \frac{v}{r} \tag{142}$$

$$mgy = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\frac{v^2}{r^2} \tag{143}$$

$$mgy = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 \tag{144}$$

$$mgy = mv^2 \tag{145}$$

$$gy = v^2 \tag{146}$$

$$v = \sqrt{gy} = \sqrt{9.8 \times 1.5} = 3.843\frac{m}{s} \tag{147}$$

(b) What is the speed of the solid cylinder when it reaches the flat?
ANSWER:

$$TE_1 = TE_2 \tag{148}$$

$$0 + 0 + mgy + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0 + 0 \tag{149}$$

$$I = \frac{1}{2}mr^2 \tag{150}$$

$$\omega = \frac{v}{r} \tag{151}$$

$$mgy = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}mr^2\frac{v^2}{r^2} \tag{152}$$

$$mgy = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \tag{153}$$

$$mgy = \frac{3}{4}mv^2 \tag{154}$$

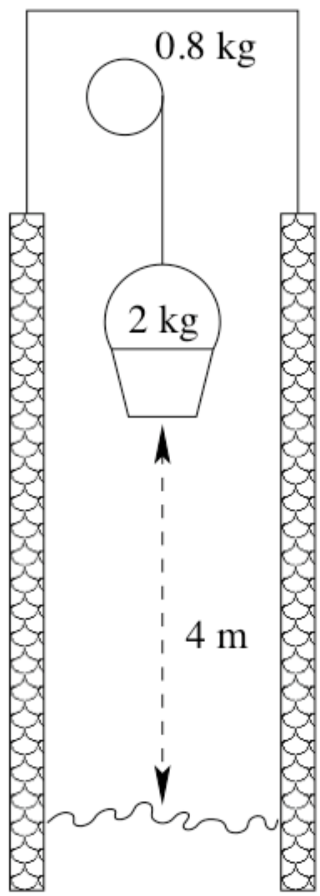
$$gy = \frac{3}{4}v^2 \tag{155}$$

$$v = \sqrt{\frac{4gy}{3}} = 4.427\frac{m}{s} \tag{156}$$

(c) Which one would win the race? **ANSWER:** Solid cylinder wins since it has smaller I!!!

Problem 8

You are lowering a bucket into a well. When the bucket is 4 m above the water, your hand slips and the bucket falls toward the water while unwinding the spindle.



(a) What is the speed of the bucket when it reaches the water?

Solution

(a) What is the speed of the bucket when it reaches the water? **ANSWER:** Conservation of Energy!

$$TE_1 = TE_2 0 + 0 + 2g4 + 0 = \frac{1}{2}2v^2 + \frac{1}{2}I\omega^2 + 0 + 0 \qquad (157)$$

$$I = \frac{1}{2}mr^2 \qquad (158)$$

$$\omega = \frac{v}{r} \qquad (159)$$

$$2g4 = \frac{1}{2}2v^2 + \frac{1}{2}(\frac{1}{2}mr^2)\frac{v^2}{r^2} \qquad (160)$$

$$2g4 = \frac{1}{2}2v^2 + \frac{1}{4}mv^2 \qquad (161)$$

$$2g4 = v^2 + \frac{1}{4}(0.8)v^2 \qquad (162)$$

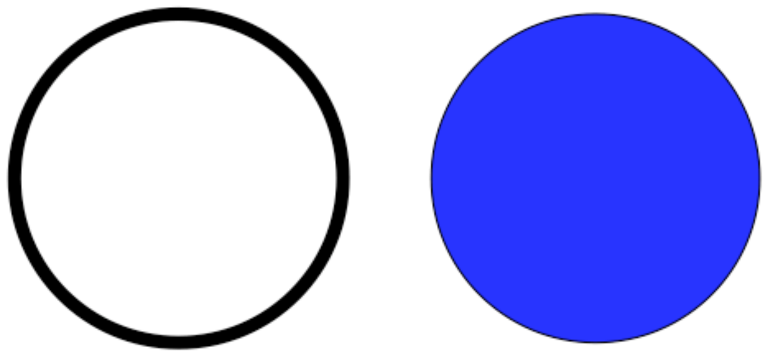
$$2g4 = 1.2v^2 \qquad (163)$$

$$8g = 1.2v^2 \qquad (164)$$

$$v = \sqrt{\frac{8g}{1.2}} = 8.08\frac{m}{s} \qquad (165)$$

Problem 9

You are in lab and see in front of you a hollow cylinder spinning with ω = 20 rpm. You decide to drop the solid cylinder on top of it.



(a) What is the final ω after you drop the solid disk?

Solution

(a) What is the final ω after you drop the solid disk? **ANSWER:** This is an angular momentum problem!

$$I_{ring} = Mr^2 = 1.5r^2 \tag{166}$$

$$I_{disk} = \frac{1}{2}Mr^2 = \frac{1}{2}2r^2 = r^2 \tag{167}$$

$$I_1 = I_{ring} = 1.5r^2 \tag{168}$$

$$I_2 = I_{ring} + I_{disk} = 2.5r^2 \tag{169}$$

$$I_1\omega_1 = I_2\omega_2 \tag{170}$$

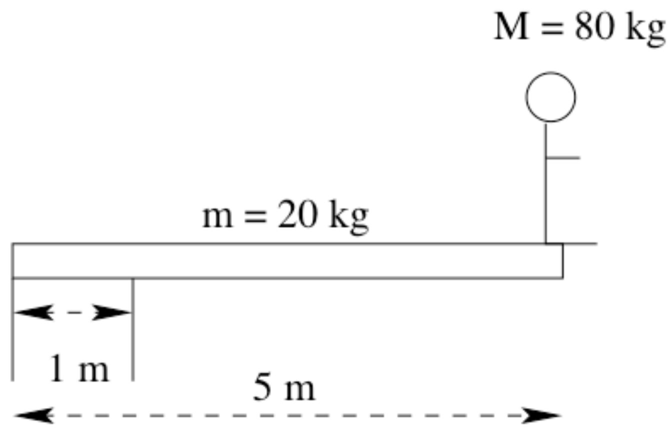
$$1.5r^2 \times (20rpm) = 2.5r^2\omega_2 \tag{171}$$

$$\frac{1.5}{2.5}(20rpm) = \omega_2 \tag{172}$$

$$\omega_2 = 12rpm \tag{173}$$

Problem 10

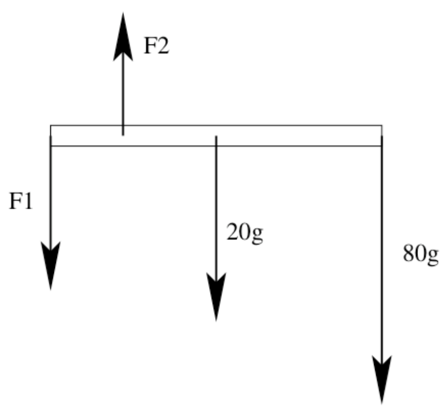
You are standing at the end of a diving board as shown in the figure below.



(a) What is the force from the "center" support?

(b) What is the force from the left-most support?

Solution



OK, this is a statics problem with one linear dimension and one rotational dimension:

$$y: \tag{174}$$
$$F_2 - F_1 - 20g - 80g = 0 \tag{175}$$
$$F_2 - F_1 - 100g = 0 \tag{176}$$

For the rotational dimension we'll take the pivot as the left end:

$$\tau: \tag{177}$$
$$1 \times F_2 - 2.5 \times 20g - 5 \times 80g = 0 \tag{178}$$
$$F_2 - 450g = 0 \tag{179}$$
$$F_2 = 450g = 4410N \tag{180}$$

Plugging into the first equation:

$$F_2 - F_1 - 100g = 0 \tag{181}$$
$$450g - F_1 - 100g = 0 \tag{182}$$
$$F_1 = 350g = 3430N \tag{183}$$

Problem 11

Three small blocks, each with mass m, are clamped at the ends and at the center of a massless rod of length L. (The rod is massless, do you think you need to include the moment of inertia of the rod?)

(a) Compute the moment of inertia of the system about an axis perpendicular to the rod and passing through a point 1/4 of the length from one end.

(b) Computer the moment of inertia of the system about at axis perpendicular to the rod and passing through the center of the rod using the "parallel axis theorem" for moment of inertia.

(c) Check that your answer is correct by explicitly calculating the moment of inertia about the axis in (b)

Solution

(a)

$$I_{L/4} = m[L/4]^2 + m[L/4]^2 + m[3L/4]^2 = (11/16)mL^2$$

(b)

$$I_{L/4} = I_{cm} + Md^2 \Longrightarrow I_{cm} = I_{L/4} - (M = 3m)(L/4)^2$$

$$I_{cm} = (11/16)mL^2 - 3m(L/4)^2 = (1/2)mL^2$$

(c)

$$I_{cm} = m(L/2)^2 + m(0)^2 + m(L/2)^2 = (1/2)mL^2$$