## Problem 1A

The Figure 1 depicts acceleration as a function of time for a short run.

a) Find the velocities at $t = 10$, 30, and 50 seconds assuming that $v(t = 0s) = 0$.

b) Draw the corresponding $v - t$ graph.

c) Find the total displacement in the 33 second run (B: 37 second run).

d) Find the average velocity for the 33 second run (B: 37 second run).

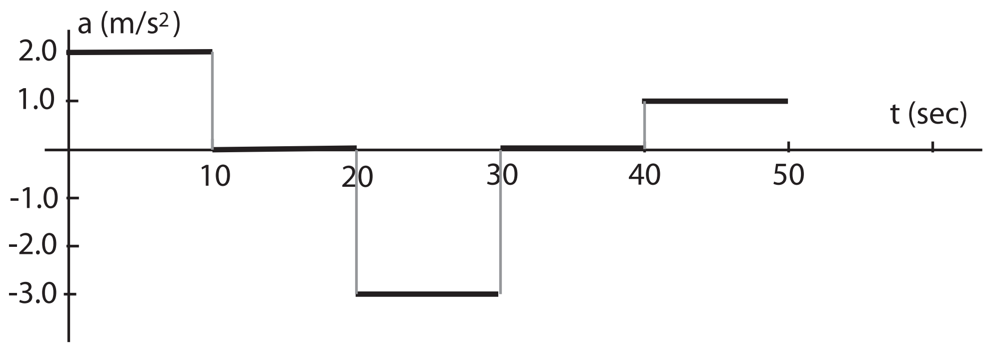


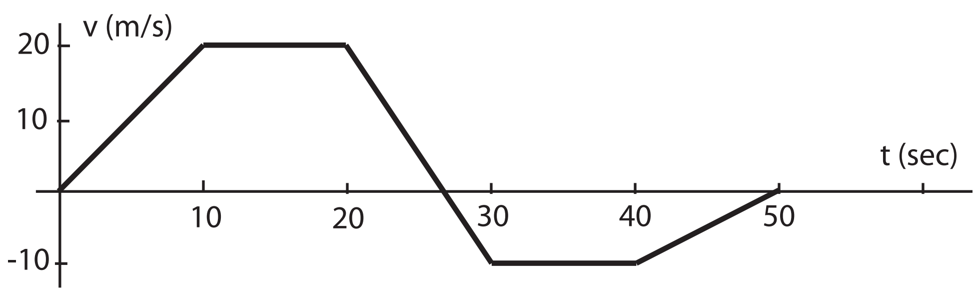

Figure 1: $a - t$ graph

## Solution

a) (7 points)

$$
\begin{aligned}
v(t = 10s) &= 0 + 2.0 \cdot 10 = 20m/s, \\
v(t = 30s) &= 20 + (-3.0) \cdot 10 = -10m/s, \\
v(t = 50s) &= -10 + 1.0 \cdot 10 = 0m/s.
\end{aligned}
$$

b) (8 points) $v - t$ graph




c) (10 points) The area under $v - t$ graph

$$\Delta x = \frac{1}{2} 10 \cdot 20 + 10 \cdot 20 + \frac{1}{2}\frac{20}{3} 20 - \frac{1}{2}\frac{10}{3} 10 - 3 \cdot 10 = 320m \qquad (B: \quad 280m)$$

d) (5 points)

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{320m}{33s} \approx 9.7m/s \qquad (B: \quad 7.6m/s)$$

## Problem 2A

A projectile is fired from the edge of the cliff with initial velocity $v_0 = 105m/s$ (B: $95m/s$) at the angle $45^0$ with horizontal. The projectile rises, then falls into the sea at the pont $P$ in 20 seconds. Find:

a) The maximal height of the projectile's trajectory (relative to the cliff).

b) The height of the cliff.

c) The horizontal distance from the cliff to the point $P$.

## Solution

We choose the positive $y$ axis in the upward direction with $y = 0$ being the coordinate of the cliff. Components of the initial velocity are

$$\begin{aligned} v_{0x} &= v_0 \cos(45^0) \approx 74.2 m/s \qquad (B: \;\; 67.2 m/s), \\ v_{0y} &= v_0 \sin(45^0) \approx 74.2 m/s \qquad (B: \;\; 67.2 m/s). \end{aligned}$$

a) (10 points)

The maximal height is reached at $t_{max}$ such that $v_{0y} - g t_{max} = 0$. $t_{max} = v_{0y}/g$. This height is

$$y_{max} = v_{0y}\, t_{max} - \frac{g t_{max}^2}{2} = \frac{v_{0y}^2}{2g} = \frac{74.2^2}{2 \cdot 9.8} \approx 281 m \qquad (B: \;\; 230 m).$$

b) (10 points)

The $y$- coordinate of the projectile at the moment when it falls into the sea is given by

$$y_P = v_{0y}\, t - \frac{g t^2}{2} = 74.2 \cdot 20 - \frac{9.8 \cdot 20^2}{2} \approx -475 m \qquad (B: \;\; 616 m).$$

Therefore, the height of the cliff

$$h \approx 475 m \qquad (B: \;\; 616 m).$$

c) (10 points)

$$L = v_{0x}\, t = 74.2 \cdot 20 \approx 1485 m \approx 1.48 km \qquad (B: \;\; 1344 m \approx 1.34 km).$$

## Problem 3

Two blocks of masses $M$ and $m$ are connected by a massless rope, which passes over a light frictionless pulley, as shown in Figure 2. The horizontal surface is rough. The coefficient of kinetic friction is $\mu_k$. Find:

a) The acceleration of each block.

b) The tension of the rope.

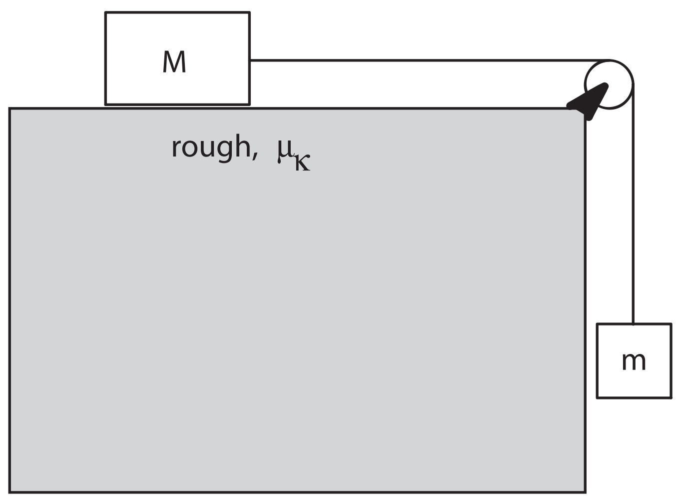
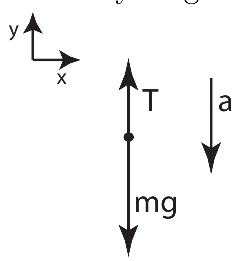



Figure 2: To problem 3.

## Solution

There are two cases: blocks moving to the right or to the left. We assume that they move to the right.

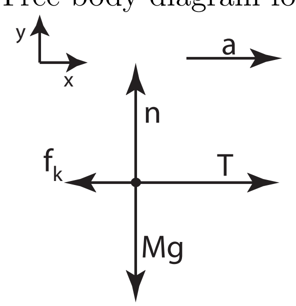
Free body diagram for block $m$:




Equations for block $m$ (only $y$ component is nontrivial)

$$mg - T = ma. \qquad (5\ points) \tag{1}$$

Free body diagram for block $M$:




Equations for block $m$

$$\begin{aligned} T - f_k &= Ma, \qquad (5\ points) \quad (2) \\ n - Mg &= 0, \qquad (5\ points) \quad (3) \end{aligned}$$

where for kinetic friction we can use

$$f_k = \mu_k n. \qquad (5\ points) \tag{4}$$

Solving the system of equations (1-4) we obtain

$$\begin{aligned} a &= \frac{m - \mu_k M}{m + M} g, \qquad (5\ points) \\ T &= \frac{mM}{m + M} g(1 + \mu_k). \qquad (5\ points) \quad (5) \end{aligned}$$