Physics 125: Classical Physics A Midterm 2 with solutions.

Problem 1A

You are studying a collision of two carts of masses $m_1 = 100$g and $m_2 = 200$g on an air track. The initial velocity of the cart 1 is $v_1 = 50$cm/s while the second cart is initially at rest. Carts collide by the ends equipped with "Velcro" strips.

a) Find a velocity of carts right after the collision.

b) What is the total impulse of the force that the cart 1 exerts on the cart 2 during the collision?

c) Estimate the magnitude of that force assuming that collision lasted about 0.05sec.

To estimate the friction force you send the cart 2 equipped with two spring bumpers at its ends along the air track with initial velocity $v_{\text{init}} = 50cm/s$. You notice that the cart bouncing from the ends of the air track is eventually stopped by friction after making 10 laps along the 2m length air track.

d) Estimate the kinetic friction force acting on the cart 2 neglecting energy losses in bumpers. Compare the value of this force with the "collision" force found in c).

Solution

a) (10 points)

$$\begin{aligned} m_1 v_1 &= (m_1 + m_2)v, \\ v &= \frac{m_1}{m_1 + m_2} v_1 = 16.7 cm/s. \quad [B: \ 33.3 cm/s] \end{aligned}$$

b) (6 points)

$$\mathcal{J} = F\Delta t = m_2 v = \frac{m_1 m_2}{m_1 + m_2} v = 3.33 \times 10^{-2} \frac{kg \cdot m}{s}.$$

c) (4 points)

$$F = \frac{\mathcal{J}}{\Delta t} = 0.67N$$

d) (10 points)

$$F_{kin} s = \frac{1}{2} m_2 v_{\text{init}}^2$$

The total distance travelled by car is $10 \times 2 = 20$m. We have

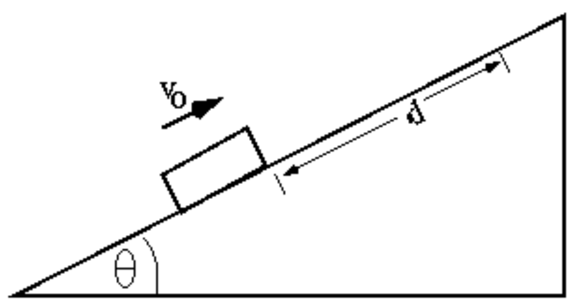
$$F_{kin} = \frac{\frac{1}{2} m_2 v_{\text{init}}^2}{s} = \frac{0.025J}{20m} = 1.25 \times 10^{-3} N \ll F = 0.67N. \quad [B: \ 6.25 \times 10^{-4} N \ll F = 0.67N.]$$

Problem 2A

A block is given an initial speed v_0 up a ramp with an incline θ. The coefficient of kinetic friction between block and ramp is μ_k.

a) In terms of v_0, μ_k, and θ find how far up the ramp the block goes ($d =?$).

b) Find d if $v_0 = 3.0\text{m/s}$, $\mu_k = 0.50$, and $\theta = 30^{\text{o}}$.



Solution

a) (20 points)

The normal force is $n = mg\cos\theta$ and the magnitude of the friction force is $f_k = \mu_k mg\cos\theta$. The final height is $d\sin\theta$.

We use conservation of energy $K_1 + U_1 + W_{fr} = K_2 + U_2$:

$$\frac{1}{2}mv_0^2 - \mu_k mg\cos\theta d = mgd\sin\theta.$$

We find

$$d = \frac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)}.$$

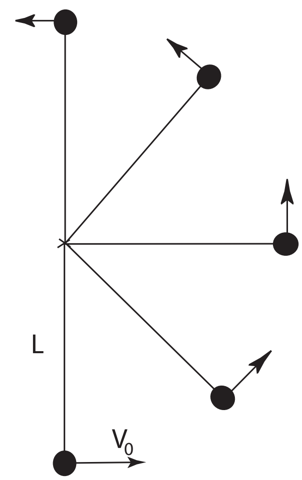
b) (10 points)

Plugging numbers into the formula for d we obtain

$$d = 0.49m. \quad [B:\ 0.24m]$$

Problem 3A

An ideal pendulum consists of a massless rigid rod of the length L and a particle of the mass m. If an initial velocity of the pendulum v_0 at the lowest point is small $v_0 < v_c$, the pendulum oscillates around its equilibrium position. If it is large $v_0 > v_c$, the pendulum rotates around the suspension point.



a) Find the "critical" velocity v_c.

b) Assume that the initial velocity is twice the critical one $v_0 = 2v_c$. What is the velocity of the pendulum in its highest position?

c) What is the angular velocity of the pendulum in the highest position?

Solution

a) (15 points) At the critical velocity pendulum stops at the highest point. We use conservation of energy

$$\frac{1}{2}mv_c^2 = mg(2L)$$

and obtain

$$v_c = \sqrt{4gL}.$$

b) (15 points) If v is the velocity at the highest point we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg(2L)$$

or

$$v = \sqrt{v_0^2 - 4gL}.$$

Substituting $v_0^2 = (2v_c)^2 = 16gL$, we obtain $v = \sqrt{12gL}$. The angular velocity at the highest point $\omega = v/L$ or

$$\omega = \sqrt{\frac{12g}{L}}.$$