Homework 14

Reading
JJS 3.1-3.3, 3.5-3.7.

Problem 1
A Hamiltonian of some system is given by

\[ H = \frac{L^2}{2I} + f L \cdot S, \]

where we assume that \( I \) and \( f \) are some constants, \( L \) is an orbital angular momentum of the system
and \( S \) is its spin operator.

a) Find the spectrum and degeneracy of eigenstates for \( f = 0 \).
b) Find the spectrum and degeneracy of eigenstates for finite \( f \).
c) Let us consider the state with given \( l \) \( (L^2 = \hbar^2 l(l+1)) \) and \( s \) \( (S^2 = \hbar^2 s(s+1)) \) and \( f = 0 \).

How will the energy level split if \( f \neq 0 \).

Problem 2
Two \( p \)-electrons are in the coupled angular momentum state \(|l_1, l_2; l, m\rangle = |1, 1; 1, -1\rangle\). If measurement is made of \( L_{1z} \) in this state, what values may be found and with what probability will these values occur?

Problem 3
Consider the function \( Y(\theta, \phi) = Y_2^0(\theta, \phi) Y_{-1}^{-1}(\theta, \phi) \). Write it down explicitly in terms of elementary functions of \( \theta \) and \( \phi \). Expand the result as a linear combination of \( Y_{l}^{m}(\theta, \phi) \).

Problem 4
Consider the operators \( a, a^\dagger, b, \) and \( b^\dagger \) obeying the following commutation relations \([a, a^\dagger] = [b, b^\dagger] = 1 \) with other commutators giving zero. We define the operators

\[ J_+ \equiv \hbar a^\dagger b, \]
\[ J_- \equiv \hbar b^\dagger a, \]
\[ J_z \equiv \frac{\hbar}{2}(a^\dagger a - b^\dagger b). \]

a) Show that the commutation relation between \( J \)-operators are the ones for the components of angular momentum.
b) Express the operator \( J^2 \equiv J_+ J_- + \frac{1}{2} (J_+ J_+ + J_- J_-) \) in terms of the operator \( N \equiv a^\dagger a + b^\dagger b \).