Homework 15

Problems with stars are not for credit and will NOT be graded.

Reading

JJS 3.4, 4.1-4.2.

Problem 1

a) Find the reduced density matrix for the first spin 1/2 in the pure state of two spin 1/2 given by the (unnormalized) state vector

$$|+\rangle \otimes |-\rangle + 2\,|-\rangle \otimes |+\rangle + i\,|+\rangle \otimes |+\rangle\,.$$

Write it down as a 2×2 matrix.

b) What are the expectation values of components of $\boldsymbol{S}$ for the first spin in this state?

c) Find the entropy (entanglement) corresponding to the found reduced density matrix.

Hint: in c) find the eigenvalues of the density matrix.

Problem 2

a) Consider a mixed ensemble of spin 1/2 systems. Suppose that the ensemble averages $[S_x]$, $[S_y]$, and $[S_z]$ are all known. Find the density matrix of this ensemble.

b) What is the condition on $[S_j]$ so that the ensemble is pure?

c) Find the entropy of this ensemble.

Problem 3

Consider an ensemble of spin 1 systems. The density matrix is now 3×3 matrix. How many independent (real) parameters are needed to characterize the density matrix? What must we know in addition to $[S_x]$, $[S_y]$, and $[S_z]$ to characterize this ensemble completely?

Problem 4

Prove that if the dynamics of an ensemble is governed by the Schrödinger equation an ensemble which is pure at $t = 0$ cannot evolve into a mixed ensemble.

Problem 5

A quantum mechanical state Ψ is known to be a simultaneous eigenstate of two Hermitian operators A and B which *anticommute*,

$$AB + BA = 0.$$

What can you say about the eigenvalues of A and B for state Ψ? Illustrate your point using the parity operator (which can be chosen to satisfy $\pi = \pi^{-1} = \pi^{\dagger}$) and the momentum operator.

*Problem 6

Consider a symmetric rectangular double-well potential:

$$V = \begin{cases} +\infty & \text{for } |x| > a+b; \\ 0 & \text{for } a < |x| < a+b; \\ V_0 > 0 & \text{for } |x| < a. \end{cases}$$

Assuming that V_0 is very high compared to the quantized energies of low-lying states, obtain an approximate expression for the energy splitting between two lowest-lying states.

Problem 7

State the selection rules for the matrix element

$$\langle \alpha'; j', m' | \, z \, | \alpha; j, m \rangle \,,$$

where z is the component of position operator, j, m are eigenvalues of J^2 and J_z respectively and α are all other quantum numbers.