Homework 18

Reading

JJS 5.3-5.6.

Problem 1

Consider a hydrogen-like atom with a single electron and the mass of the nucleus M and charge Ze.

a) Write down the Hamiltonian for the nucleus-electron system.

b) Change radius vectors to the radius vector of the center of mass and to the position of the electron relative to the nucleus. Show that center of mass motion is separated and that the central potential problem depends only on the reduced mass of the system.

Problem 2

The nucleus of the atom is not point-like. The problem is to find the energy shift of the ground state of the hydrogen-like atom due to the finite size of the nucleus as compared to the energy of the ground state in the Coulomb potential.

a) Approximate the nucleus of a hydrogen-like atom as a uniformly charged sphere of the radius R and charge Ze. Find the electrostatic potential of the electron both for $r > R$ and for $r < R$.

b) Consider the difference between the potential obtained in a) and the Coulomb potential $V_0(r) = -\frac{Ze^2}{r}$ as a perturbation. Calculate the shift of the energy of the ground state of the hydrogen-like atom due to this perturbation in the first order perturbation theory.

c) Compare the shift obtained in b) with the shifts coming from relativistic corrections for $Z = 1$.

Hint: In b) use the known wave function of the ground state of a hydrogen-like atom.

Problem 3

A spin 1/2 particle with magnetic moment μ is in the uniform magnetic field $\boldsymbol{B}(t)$ of the form

$$B_x = B_0 \cos \omega_0 t, \qquad B_y = B_0 \sin \omega_0 t, \qquad B_z = B_1,$$

where B_0, B_1 and ω_0 are constants.

At $t = 0$ the particle was in the state with $s_z = 1/2$. Find the absorption power $P(t)$ as a function of time (the work done by an external field per unit time). Discuss, in particular, the case $B_0/B_1 \ll 1$. Pay attention to a resonance character of the absorption power.

Hint: Read JJS 5.5.

Problem 4

Show that the Taylor expansion in λ of $W = e^{A+\lambda B}$, where A and B are some non-commuting matrices is given by

$$W = e^A \left[1 + \lambda \int_0^1 d\tau_1 \; B_I(\tau_1) + \ldots \right],$$

where

$$B_I(\tau) = e^{-\tau A} B e^{\tau A}.$$

Write explicitly the second order term in λ.

Hint: Consider the quantity $U(\tau) = e^{-\tau A} e^{\tau(A+\lambda B)}$, write down the differential equation for this quantity differentiating it with respect to τ and solve it by series expansion.

Problem 5

A uniform electric field $\mathcal{E}$ is **suddenly** turned on along the axis of a charged linear harmonic oscillator which was in the ground state. Find the probabilities of exciting of different eigenstates of the oscillator after turning on the field.