Homework 19

Reading
JJS 5.6.

Problem 1
A particle is in the ground state of an infinite potential well of the width $a$ at $t \to -\infty$. In addition it is in the weak uniform time-dependent potential of the form

$$V(x, t) = -xF_0 e^{-t^2/\tau^2}.$$ 

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.

Problem 2
A linear harmonic oscillator is in the $n$-th eigenstate at $t \to -\infty$. It is subjected to a uniform electric field of the form

$$E(t) = E_0 e^{-t^2/\tau^2}.$$ 

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.

Problem 3
In the previous problem find in the second order of perturbation theory the probabilities of transitions forbidden in the first order of perturbation theory. Compare probabilities $W(n \to n \pm 2)$ with $W(n \to n \pm 1)$.

Problem 4
A particle is in the ground state of the potential $U(x) = -\alpha \delta(x)$ for $t < 0$. A weak uniform field $V(x, t) = -xF_0 \sin \omega_0 t$ is applied to the system for $t > 0$. Find the probability $W_0(t)$ that the particle is still in the ground state at time $t$. Consider only the case $\hbar \omega_0 \gg |E_0|$, where $E_0$ is the bound state energy.

Hints: (i) For energies $E \gg |E_0|$ one can use the wave functions of free particle without potential as the effect of the potential on high energy states is small. (ii) First calculate $w$, the rate of exciting the particle from the ground state per unit time. (iii) Use Fermi’s Golden Rule.