Homework 19

Reading

JJS 5.6.

Problem 1

A particle is in the ground state of an infinite potential well of the width a at $t \to -\infty$. In addition it is in the weak uniform time-dependent potential of the form

$$V(x,t) = -xF_0\, e^{-t^2/\tau^2}.$$

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.

Problem 2

A linear harmonic oscillator is in the n-th eigenstate at $t \to -\infty$. It is subjected to a uniform electric field of the form

$$\mathcal{E}(t) = \mathcal{E}_0\, e^{-t^2/\tau^2}.$$

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.

Problem 3

In the previous problem find in the second order of perturbation theory the probabilities of transitions forbidden in the first order of perturbation theory. Compare probabilities $W(n \to n \pm 2)$ with $W(n \to n \pm 1)$.

Problem 4

A particle is in the ground state of the potential $U(x) = -\alpha\delta(x)$ for $t < 0$. A weak uniform field $V(x,t) = -xF_0 \sin\omega_0 t$ is applied to the system for $t > 0$. Find the probability $W_0(t)$ that the particle is still in the ground state at time t. Consider only the case $\hbar\omega_0 \gg |E_0|$, where E_0 is the bound state energy.

Hints: (i) For energies $E \gg |E_0|$ one can use the wave functions of free particle without potential as the effect of the potential on high energy states is small. (ii) First calculate w - the rate of exciting the particle from the ground state per unit time. (iii) Use Fermi's Golden Rule.