Homework 20

Reading
Merzbacher Ch. 7., LL 45.

Problem 1
Determine the energy level in a one-dimensional potential well whose depth is small. Namely, $|U| \ll \frac{\hbar^2}{ma^2}$, where $U$ is the typical depth and $a$ is the width of the potential.

*Hint:* Use the whole potential energy as a time-independent perturbation. Read LL 45.

Problem 2
Use the WKB approximation to find the allowed energies of the general power law potential $V(x) = \alpha |x|^\nu$, where $\nu$ is a positive number. Check your result for the case $\nu = 2$.

Problem 3
In quasiclassical approximation find the energy levels in a one-dimensional potential $U(x) = \alpha x^4$. For which energy levels you can trust the obtained result?

Problem 4
In quasiclassical approximation find the transmission coefficient of a one-dimensional potential barrier $U(x) = U_0(1 - x^2/a^2)$ for $|x| < a$ and zero otherwise. For which energies you can trust the obtained result?

Problem 5
Show that for the potential of the form

\[ U(x) \]

the Bohr-Sommerfeld quantization condition has a form

\[ \int_0^{x_0} dx \sqrt{2m(E - U(x))} = \pi \hbar \left(n + \frac{3}{4}\right). \]