Homework 22

Reading
JJS Ch. 6, 7.1-7.2

Problem 1
Two identical particles of mass \( m \) interact with each other via the three-dimensional harmonic potential \( V(r_1 - r_2) = k(r_1 - r_2)^2/2 \). Find the energy spectrum and the eigenfunctions of the system, when the particles are
(i) bosons,
(ii) fermions.
Assume that fermions are “spinless” (i.e., spin-polarized).

Problem 2
\( N \) identical spin 1/2 particles are subjected to a one-dimensional simple harmonic oscillator potential. What is the ground state energy? What is the Fermi energy?

Problem 3
Three spin 0 particles are situated at the corners of an equilateral triangle. Let us define the \( z \)-axis to go through the center and in the direction normal to the plane of the triangle. The whole system is free to rotate about the \( z \)-axis. Using statistics considerations, obtain restrictions on the magnetic quantum numbers corresponding to \( J_z \).

Problem 4
In Born approximation find the amplitude of scattering and the total cross section in the field
\[
U(r) = \alpha \delta(r - R).
\]
Here \( \alpha, R \) are some constants, the potential is a function of the distance to the center \( r \) only and the delta-function is a 1D delta-function.
Study the cases of small and large energies and applicability of the approximation.

Hint: The integral for the total cross section \( \sigma(E) \) cannot be taken in terms of elementary functions. Simplify it and leave it as it is. In the limiting cases of small and large \( E \) calculate the integral approximately.