Homework 22

Reading

JJS Ch. 6, 7.1-7.2

Problem 1

Two identical particles of mass m interact with each other via the three-dimensional harmonic potential $V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = k(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2/2$. Find the energy spectrum and the eigenfunctions of the system, when the particles are

(i) bosons,

(ii) fermions.

Assume that fermions are "spinless" (i.e., spin-polarized).

Problem 2

N identical spin 1/2 particles are subjected to a one-dimensional simple harmonic oscillator potential. What is the ground state energy? What is the Fermi energy?

Problem 3

Three spin 0 particles are situated at the corners of an equilateral triangle. Let us define the z-axis to go through the center and in the direction normal to the plane of the triangle. The whole system is free to rotate about the z-axis. Using statistics considerations, obtain restrictions on the magnetic quantum numbers corresponding to J_z.

Problem 4

In Born approximation find the amplitude of scattering and the total cross section in the field

$$U(r) = \alpha\delta(r - R).$$

Here α, R are some constants, the potential is a function of the distance to the center r only and the delta-function is a 1D delta-function.

Study the cases of small and large energies and applicability of the approximation.

Hint: The integral for the total cross section $\sigma(E)$ cannot be taken in terms of elementary functions. Simplify it and leave it as it is. In the limiting cases of small and large E calculate the integral approximately.