Homework 14

Reading

JJS 3.4, 4.1-4.2.

Problem 1

Consider an ensemble of spin 1 systems. The density matrix is now 3×3 matrix. How many independent (real) parameters are needed to characterize the density matrix? What must we know in addition to $[S_x]$, $[S_y]$, and $[S_z]$ to characterize this ensemble completely?

Problem 2

Prove that if the dynamics of an ensemble is governed by the Schrödinger equation an ensemble which is pure at $t = 0$ cannot evolve into a mixed ensemble.

Problem 3

A particle is in the state $\psi(x) = \sqrt{\frac{2}{\pi}} \sin(x)$ in a 1d box $x \in [0, \pi]$. Let us consider the half of the box $x \in [0, \pi/2]$ as a subsystem. What is the reduced density matrix of the particle for this subsystem? What is the entropy calculated for this reduced density matrix?

Problem 4

A quantum mechanical state Ψ is known to be a simultaneous eigenstate of two Hermitian operators A and B which *anticommute*,

$$AB + BA = 0.$$

What can you say about the eigenvalues of A and B for state Ψ? Illustrate your point using the parity operator (which can be chosen to satisfy $\pi = \pi^{-1} = \pi^\dagger$) and the momentum operator.

*Problem 5

Consider a symmetric rectangular double-well potential:

$$V = \begin{cases} +\infty & \text{for } |x| > a + b; \\ 0 & \text{for } a < |x| < a + b; \\ V_0 > 0 & \text{for } |x| < a. \end{cases}$$

Assuming that V_0 is very high compared to the quantized energies of low-lying states, obtain an approximate expression for the energy splitting between two lowest-lying states.