Homework 16

Reading

JJS 5.1-5.2.

Problem 1

Consider the following terms of some Hamiltonians.

$$A)\ \frac{\boldsymbol{p}^2}{2m}, \qquad B)\ \boldsymbol{D}\cdot(\boldsymbol{L}+2\boldsymbol{S}), \qquad C)\ V(r),$$
$$D)\ V(r)\boldsymbol{L}^2, \qquad E)\ \boldsymbol{r}\cdot(\boldsymbol{p}\times\boldsymbol{J}), \qquad F)\ \boldsymbol{L}\cdot\boldsymbol{S}$$

in obvious notations (here $\boldsymbol{D}$ is some fixed vector which does not transform).

a) List parity even terms.

b) List parity odd terms.

c) List time-reversal even terms.

d) List time-reversal odd terms.

Problem 2

For the particle in an infinite one-dimensional potential well of the width a ($0 < x < a$), find in the first order of perturbation theory the energy shifts under perturbation of the type:

a) $V(x) = \frac{V_0}{a}(a - |2x - a|)$;

b) $V(x) = V_0\, \theta_{b<x<a-b}$.

State the conditions of applicability of obtained results.

Remark: The step function $\theta_{c<x<d}$ is equal to 1 for $c < x < d$ and 0 everywhere else.

Problem 3

Show that the energy shift $\Delta_n^{(1)}$ obtained in the first order of perturbation theory for the previous problem but for an **arbitrary** perturbation $V(x)$ does not depend on n for sufficiently large n.

Problem 4

Let us write down the Hamiltonian for a harmonic oscillator as

$$H = \frac{p^2}{2m} + \frac{kx^2}{2} + \frac{\alpha x^2}{2}.$$

Consider formally the last term $\alpha x^2/2$ as a perturbation. Using perturbation theory up (and including) to the second order calculate the shifts of energy levels. Compare the result with the exact answer. What is the condition for the convergence of perturbation series?

Problem 5

For the problem 1, take $V(x) = \alpha\, \delta(x - a/2)$ and find the shifts of energy levels up to (and including) the second order of perturbation series. State the conditions of applicability of the obtained result.