Homework 17

Reading

JJS 5.3-5.6.

Problem 1

The plane rotator with the moment of inertia I and electric dipole moment $\boldsymbol{d}$ is placed in a uniform electric field $\boldsymbol{\mathcal{E}}_0$ lying in the plane of rotation. Considering the effect of the field as perturbation, find the polarizability of the ground state of a rotator. State the conditions of applicability of the obtained result.

Remark: the definition of the polarizability is $\alpha \equiv \partial |\langle \boldsymbol{d} \rangle| / \partial \mathcal{E}_0$.

Problem 2

Find approximate wave functions of the plane rotator in a uniform electric field (for the system from the previous problem). Do it in the first order in electric field of the perturbation theory.

Problem 3

Consider a hydrogen-like atom with a single electron and the mass of the nucleus M and charge Ze.

a) Write down the Hamiltonian for the nucleus-electron system.

b) Change radius vectors to the radius vector of the center of mass and to the position of the electron relative to the nucleus. Show that center of mass motion is separated and that the central potential problem depends only on the reduced mass of the system.

Problem 4

The nucleus of the atom is not point-like. The problem is to find the energy shift of the ground state of the hydrogen-like atom due to the finite size of the nucleus as compared to the energy of the ground state in the Coulomb potential.

a) Approximate the nucleus of a hydrogen-like atom as a uniformly charged sphere of the radius R and charge Ze. Find the electrostatic potential of the electron both for $r > R$ and for $r < R$.

b) Consider the difference between the potential obtained in a) and the Coulomb potential $V_0(r) = -\frac{Ze^2}{r}$ as a perturbation. Calculate the shift of the energy of the ground state of the hydrogen-like atom due to this perturbation in the first order perturbation theory.

c) Compare the shift obtained in b) with the shifts coming from relativistic corrections for $Z = 1$.

Hint: In b) use the known wave function of the ground state of a hydrogen-like atom.