Homework 18

Reading

JJS 5.5, 5.6.

Problem 1

A spin 1/2 particle with magnetic moment μ is in the uniform magnetic field $\boldsymbol{B}(t)$ of the form

$$B_x = B_0 \cos\omega_0 t, \qquad B_y = B_0 \sin\omega_0 t, \qquad B_z = B_1,$$

where B_0, B_1 and ω_0 are constants.

At $t = 0$ the particle was in the state with $s_z = 1/2$. Find the absorption power $P(t)$ as a function of time (the work done by an external field per unit time). Discuss, in particular, the case $B_0/B_1 \ll 1$. Pay attention to a resonance character of the absorption power.

Hint: Read JJS 5.5.

Problem 2

Show that the Taylor expansion in λ of $W = e^{A+\lambda B}$, where A and B are some non-commuting matrices is given by

$$W = e^A \left[1 + \lambda \int_0^1 d\tau_1 \; B_I(\tau_1) + \ldots\right],$$

where

$$B_I(\tau) = e^{-\tau A} B e^{\tau A}.$$

Write explicitly the second order term in λ.

Hint: Consider the quantity $U(\tau) = e^{-\tau A} e^{\tau(A+\lambda B)}$, write down the differential equation for this quantity differentiating it with respect to τ and solve it by series expansion.

Problem 3

A uniform electric field $\mathcal{E}$ is **suddenly** turned on along the axis of a charged linear harmonic oscillator which was in the ground state. Find the probabilities of exciting of different eigenstates of the oscillator after turning on the field.

Problem 4

A particle is in the ground state of an infinite potential well of the width a at $t \to -\infty$. In addition it is in the weak uniform time-dependent potential of the form

$$V(x,t) = -xF_0\, e^{-t^2/\tau^2}.$$

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.