Homework 19

Reading

Merzbacher Ch. 7., **LL** 45, **JJS** 5.6

Problem 1

A linear harmonic oscillator is in the n-th eigenstate at $t \to -\infty$. It is subjected to a uniform electric field of the form

$$\mathcal{E}(t) = \mathcal{E}_0 \, e^{-t^2/\tau^2}.$$

a) Calculate in the first order of perturbation theory the probabilities of exciting the particle to different states at $t \to +\infty$.

b) State the condition of applicability of the result.

Problem 2

In the previous problem find in the second order of perturbation theory the probabilities of transitions forbidden in the first order of perturbation theory. Compare probabilities $W(n \to n \pm 2)$ with $W(n \to n \pm 1)$.

Problem 3

A particle is in the ground state of the potential $U(x) = -\alpha\delta(x)$ for $t < 0$. A weak uniform field $V(x,t) = -xF_0 \sin\omega_0 t$ is applied to the system for $t > 0$. Find the probability $W_0(t)$ that the particle is still in the ground state at time t. Consider only the case $\hbar\omega_0 \gg |E_0|$, where E_0 is the bound state energy.

Hints: (i) For energies $E \gg |E_0|$ one can use the wave functions of free particle without potential as the effect of the potential on high energy states is small. (ii) First calculate w - the rate of exciting the particle from the ground state per unit time. (iii) Use Fermi's Golden Rule.

Problem 4

Determine the energy level in a one-dimensional potential well whose depth is small. Namely, $|U| \ll \frac{\hbar^2}{ma^2}$, where U is the typical depth and a is the width of the potential.

Hint: Use the whole potential energy as a time-independent perturbation. Read **LL** 45.

Problem 5

Use the WKB approximation to find the allowed energies of the general power law potential $V(x) = \alpha|x|^\nu$, where ν is a positive number. Check your result for the case $\nu = 2$.

Problem 6

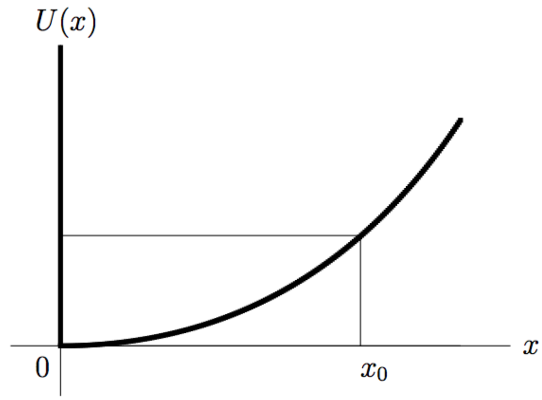
In quasiclassical approximation find the energy levels in a one-dimensional potential $U(x) = \alpha x^4$. For which energy levels you can trust the obtained result?

*Problem 7

In quasiclassical approximation find the transmission coefficient of a one-dimensional potential barrier $U(x) = U_0(1 - x^2/a^2)$ for $|x| < a$ and zero otherwise. For which energies you can trust the obtained result?

Problem 8

Show that for the potential of the form



the Bohr-Sommerfeld quantization condition has a form

$$\int_0^{x_0} dx\, \sqrt{2m(E - U(x))} = \pi\hbar\Big(n + \frac{3}{4}\Big).$$