Homework 20

Reading

Merzbacher Ch. 7., **LL** 45, **JJS** Ch. 6.

Problem 1

A particle stays in the bound state of the energy E. Using the Bohr-Sommerfeld quantization condition, find the change of the energy of the state δE when the potential $U(x)$ changes by a small variation $\delta U(x)$. Interpret the obtained result in terms of classical mechanics.

Problem 2

Find the probability of the reflection above the barrier of the form

$$V(x) = V_0 \frac{a^2}{x^2 + a^2}$$

for $E \gg V_0$ using quasiclassical approximation.

Problem 3

The transition amplitude or propagator is given by path integral

$$\langle x_f, t_f | x_i, 0 \rangle = \int \mathcal{D}x \; e^{\frac{i}{\hbar} \int dt \, \frac{m\dot{x}^2}{2}},$$

where the summation is taken over trajectories satisfying $x(0) = x_i$ and $x(t_f) = x_f$.

Perform the integration over paths and find the explicit form of the propagator.

Hint:
1) write $x(t) = x_i + \frac{x_f - x_i}{t_f} t + \sum_{n=1}^{\infty} q_n \sin\left(\pi n \frac{t}{t_f}\right)$ where first two terms give the solution of classical equations of motion and q_n are coordinates parametrizing fluctuations around the classical solution.

2) Substitute the above parametrization into the path integral, use the measure as $\mathcal{D}x = \prod_{n=1}^{\infty} dq_n$ and perform the integration.