Physics 556: Solid State Physics II

Homework 4

Read: **FW** 9, 10, 11, 12

"**FW**" refers to Fetter, Walecka book.
Problems with stars are not for credit and will NOT be graded.

Exercise 1: Uniform magnetic field

Consider a non-interacting Fermi gas in a uniform external magnetic field B at $T = 0$. Keeping only Zeeman part of an interaction with magnetic field (and neglecting orbital effects of magnetic field) we write the interaction part of the Hamiltonian as

$$H_B = \omega_B \sum_p (c^\dagger_{p,\uparrow} c_{p,\uparrow} - c^\dagger_{p,\downarrow} c_{p,\downarrow}), \tag{1}$$

where $\omega_B = \mu B$ - Zeeman energy.

a) Describe the exact ground state of non-interacting fermions (with spin 1/2) in a uniform magnetic field.

b) What is the exact Green's function $G^B_{\alpha\beta}(k, \omega)$ of such a system if the chemical potential is μ?

c) Find the magnetization $M = \langle c^\dagger_\uparrow c_\uparrow - c^\dagger_\downarrow c_\downarrow \rangle$ of the ground state using the obtained form of $G^B(k, \omega)$. Express the result as a function of the density of the gas.

d) What is the susceptibility $\chi_{Pauli} = \partial M / \partial B$ of the non-interacting gas.

Exercise 2: Uniform magnetic field

It is instructive to obtain the Green's function $G^B_{\alpha\beta}$ of the previous problem starting from the Green's function of a non-interacting gas without an external magnetic field $G^0_{\alpha\beta}$. Considering the part of the hamiltonian (1) as an interaction part, construct the perturbation theory in magnetic field. Calculate the sum (geometric series) of all diagrams for G^B and obtain the exact expression for G^B. Calculate the sum using diagrammatic methods.

Exercise 3: Proper self-energy to the order 3

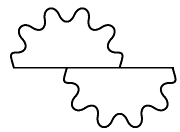
Draw all diagrams for the proper self-energy in first, second, and third orders of perturbation theory in two-particle interactions. You can omit spin indices and labels of vertices and edges. Do not forget to draw arrows on electronic Green's functions. Try to group diagrams according to some principles (an attempt to classification). Please, draw diagrams as neatly as possible.

Exercise 4: A diagram for the self-energy

Consider the diagram shown in the figure as a diagram in momentum representation for $\Sigma^{\star}_{\alpha\beta}(k)$.

a) Place spin indices and 4-momenta on vertices and edges of the diagram.

b) Write down an analytic expression for the contribution to the self-energy corresponding to the diagram.



Exercise 5: Proper polarization to the order 2

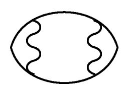
Draw all diagrams for the proper polarization in zeroth, first, and second orders of perturbation theory in two-particle interactions. You can omit spin indices and labels of vertices and edges. Do not forget to draw arrows on electronic Green's functions. Try to group diagrams according to some principles (an attempt to classification). Please, draw diagrams as neatly as possible.

Exercise 6: A diagram for the proper polarization

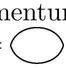
Consider the diagram shown in the figure as a diagram in momentum representation for $\Pi^{\star}_{\alpha\beta}(k)$.

a) Place spin indices and 4-momenta on vertices and edges of the diagram.

b) Write down an analytic expression for the contribution to the polarization corresponding to the diagram.



Exercise 7: A zeroth order polarization

Using momentum representation calculate the zeroth order contribution to the polarization $\Pi^0(k,\omega) =$  of an electron gas. Find explicitly also $\operatorname{Im}\Pi^0(k,\omega)$. Do calculations in

a) One dimension.

*b) Two dimensions.

*c) Three dimensions.

Hint: For three-dimensional calculation see FW Ch 12, p. 158.