Physics 556: Solid State Physics II

Homework 6

Read: FW 13-16, 4-5

"**FW**" refers to Fetter, Walecka book.
Problems with stars are not for credit and will NOT be graded.

Exercise 1: Density-density response function in $d = 1$

Consider one-dimensional Fermi gas in an external field

$$H_{int} = -\int dx\, e\phi(x,t)\hat{n}(x,t).$$

The linear response of the density to the field $e\phi(x,t)$ can be written in Fourrier representation as

$$\langle \hat{n}(k,\omega)\rangle = Q(k,\omega)e\phi(k,\omega).$$

Find the response function $Q(k,\omega)$ for small $|k| \ll k_F$ and $|\omega| \ll \epsilon_F/\hbar$.

Exercise 2: Plasmon in two dimensions

Derive the dispersion of plasma oscillations $\Omega(q)$ for small q in two dimensions in RPA approximation. Assume a jellium model of a 2d electron gas with particle density n, particle mass m and 3d Coulomb interaction $V_0(x) = e^2/x$.

Exercise 3: Two-particle correlations (FW 5.10)

Two-particle correlations in the ground state can be characterized by the function

$$g(x,y) \equiv \langle \Psi_0|\hat{\psi}^\dagger_\alpha(x)\hat{\psi}^\dagger_\beta(y)\hat{\psi}_\beta(y)\hat{\psi}_\alpha(x)|\Psi_0\rangle - \langle \Psi_0|\hat{\psi}^\dagger_\alpha(x)\hat{\psi}_\alpha(x)|\Psi_0\rangle\langle \Psi_0|\hat{\psi}^\dagger_\beta(x)\hat{\psi}_\beta(x)|\Psi_0\rangle.$$

Show that for a non-interacting spin-$\frac{1}{2}$ Fermi gas

$$g^{(0)}(|x-y|) = -\frac{1}{2}\rho_0^2\left[\frac{3j_1(k_F|x-y|)}{k_F|x-y|}\right]^2,$$

where $j_1(x)$ is a spherical Bessel function.

Exercise 4: Ultrarelativistic gas (FW 2.2)

Given the energy spectrum $\epsilon_p = \sqrt{(pc)^2 + m_0^2c^4} \to pc$ ($p \to \infty$), prove that an ultrarelativistic ideal gas satisfies the equation of state $PV = E/3$ where E is the total energy.

Exercise 5: BEC in 2d (FW 2.3)

Show that there is no Bose-Einstein condensation at any finite temperature for a two-dimensional ideal Bose gas with spectrum $\epsilon_p^0 = \frac{p^2}{2m}$. What happens if one considers the ideal gas but with a modified spectrum $\epsilon_p = \sqrt{\epsilon_p^0\left(\epsilon_p^0 + 2mc^2\right)}$? This modification can occur as a result of interactions between bosons.

Exercise 6: Paramagnetic spin susceptibility (FW 2.6)

Prove that the paramagnetic spin susceptibility of a free Fermi gas of spin-$\frac{1}{2}$ particles at $T = 0$ is given by $\chi(T = 0) = \frac{3}{2}\frac{\mu_0^2}{\epsilon_F}\frac{N}{V}$ where μ_0 is the magnetic moment of one of the particles. Derive the corresponding high-temperature result $\chi(T \to \infty) = \frac{\mu_0^2}{k_B T}\frac{N}{V}$.