1. **Gravitational potential energy.** A rocket of mass \( m = 440. \text{kg} \) is launched from the surface of the earth. The earth’s radius is \( R = 6.37 \times 10^6 \text{m} \), the earth’s mass is \( M = 5.98 \times 10^{24} \text{kg} \), and Newton’s constant \( G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \). When appropriate, you may use the approximate gravitational force per unit mass \( g = 9.8 \text{N/kg} \) (near earth’s surface).

   a. To 1% accuracy, how much work must be done (against gravity) to raise the rocket by \( h = 2.00 \text{km} = 2.00 \times 10^3 \text{m} \) above the earth’s surface?

   \[
   W = -W_{\text{grav}} = \Delta U_{\text{grav}} = mgh = (440 \times 10^3 \text{kg})(9.8 \text{m/s}^2)(2.00 \times 10^3 \text{m}) = 8.6 \times 10^9 \text{J}
   \]

   b. Explain in words why it takes less than 1000 times this work to raise the rocket to \( h' = 1000h \). If force were constant, work would be proportional to \( h \).

   However, the force of gravity diminishes as \( h \) increases, so work is less.

   c. How much work must be done against gravity to raise the rocket to \( h = 2.00 \times 10^6 \text{m} \) above the earth’s surface?

   \[
   W = \Delta U_{\text{grav}} = - \frac{GMm}{R+h} + \frac{GMm}{R} = mg\left(\frac{R^2}{R+h}\right) = 8.6 \times 10^9 \text{J} \times \frac{6.37}{8.37} = 6.6 \times 10^9 \text{J}
   \]

2. **Collision in one dimension.** A mass \( m_1 = 1.00 \text{kg} \) moves with velocity \( v_1 = +3.00 \text{m/s} \). Positive is to the right. It collides with a mass \( m_2 = 2.00 \text{kg} \), and rebounds with velocity \( v_1' = -1.00 \text{m/s} \).

   a. What is the velocity \( v_2' \) of \( m_2 \) after the collision?

   \[
   m_1v_1 = m_1v_1' + m_2v_2'
   \]

   \[
   \frac{m_2}{m_1} = 2; \quad v_2' = \frac{1}{2}(v_1 - v_1') = \frac{1}{2}(30 + 10) = 20 \text{m/s}
   \]

   b. How much work did \( m_2 \) do on \( m_1 \) during this collision?

   \[
   \text{Work} = \frac{\Delta \text{KE}_1}{2} = \frac{1}{2}m_1(v_1'^2 - v_1^2) = \frac{1}{2}(1.00 \text{kg})(20 \text{m/s})^2 = 200 \text{J}
   \]

   c. How much work did \( m_1 \) do on \( m_2 \) during this collision?

   \[
   \text{Work}_{12} = \frac{\Delta \text{KE}_2}{2} = \frac{1}{2}m_2(v_2'^2 - v_2^2) = \frac{1}{2}(2.00 \text{kg})(20 \text{m/s})^2 = 400 \text{J}
   \]

   d. How much total work was done during the collision?

   \[
   W_{\text{tot}} = 0 \quad \text{[The collision was elastic!]} \]

3. A block of mass \( m = 2.6 \text{kg} \) slides on level ground with coefficient of kinetic friction \( \mu = 0.35 \).

   a. How much work does friction do during \( x = 1.00 \text{m} \) of slide?

   \[
   W_f = -\mu mgx \quad \text{since } N = mg \text{ and } \theta = 180^\circ \quad W = -8.9 \text{J}
   \]

   b. How much initial velocity \( v_0 \) should the block have, if it is to slide \( x = 7.0 \text{m} \) before coming to rest?

   Friction does \(-7 \times 8.9 \text{J} = -62.4 \text{J} = \Delta \text{KE} = 0 - (\frac{1}{2}mV^2)\). \[
   \frac{1}{2}mV^2 = 62.4 \text{J} \quad \text{so } V = 6.9 \text{m/s}
   \]