1. **Inelastic collision in one dimension.** Two railroad cars (shown above) with \( M_1 = M_2 = 1.2 \times 10^4 \text{ kg} \) travel with equal and opposite velocities on frictionless parallel tracks. Car 1 travels in the \( \hat{i} \) direction with velocity \( \vec{v}_1 = 25\hat{i} \text{ m/s} \).

   a. What is the momentum \( \vec{p}_1 \) of car 1? 
   \[ M_1 \vec{v}_1 = 3 \times 10^5 \text{ kg m/s} \hat{i} \]

   b. What is the momentum \( \vec{p}_2 \) of car 2? 
   \[ M_2 \vec{v}_2 = -M_1 \vec{v}_1 = -3 \times 10^5 \text{ kg m/s} \hat{i} \]

   c. What is the total momentum \( \vec{p}_{\text{tot}} \)? 
   \[ \vec{p}_1 + \vec{p}_2 = 0 \]

A parcel of mass \( \Delta M = 0.20M_1 = 2.4 \times 10^3 \text{ kg} \) is disconnected without friction from car 1. In other words, it is on wheels and rolls gently off car 1, landing on car 2.

   d. The velocity \( \vec{v}_1' = \vec{v}_1 \) is unchanged. What would have to be different in order for \( \vec{v}_1' \neq \vec{v}_1 \)? 
   If the parcel had been shoved off car 1.
   Even then, \( \vec{p}_1' + \vec{p}_2' = 0 \) (momentum still conserved)

   e. The mass \( \Delta M \) slides on car 2 until friction brings the parcel to rest on the car.

Find the unknown velocity \( \vec{v}_2' \)

\[
(M_1 - \Delta M) \vec{v}_1' + (M_2 + \Delta M) \vec{v}_2' = M_1 \vec{v}_1 + M_2 \vec{v}_2 = 0
\]

\[
\vec{v}_2' = -\left( \frac{M_1 - \Delta M}{M_2 + \Delta M} \right) \vec{v}_1' = -\left( \frac{1.2}{1.2 + 0.2} \right) \vec{v}_1 = \vec{v}_1 
\]

\[
\vec{v}_2' = \vec{v}_1 = -25 \hat{i} \text{ m/s}
\]

\[17 \text{ m/s}\]
2. **Elastic collision in two dimensions.** A mass $M_1 = 0.35\ \text{kg}$ travels on a frictionless horizontal surface at velocity $\vec{v}_1 = v_{1x}\hat{i} = 2.5\ \text{m/s}$. It hits a stationary ($\vec{v}_2 = 0$) frictionless mass $M_2 = 2M_1$ and rebounds with velocity $\vec{v}_1' = v_{1x}'\hat{i} + v_{1y}'\hat{j}$ where the magnitude $|\vec{v}_1'| = v_{1y}$ is unknown. The mass $M_2$ recoils with unknown magnitude and direction $\vec{v}_2 = v_{2x}'\hat{i} + v_{2y}'\hat{j}$. Notice that this problem has three unknowns.

a. What is the total momentum of the system?

$$P_{\text{tot}} = M_1 \vec{v}_1 + 0 = 0.875\ \text{kg}\cdot\text{m/s}.$$

b. Write an equation relating $v_{2x}'$ to other velocity components like $v_{1x}$ and $v_{1y}$.

$$P_{\text{final}} = 2M_1 v_{2x}' = P_{\text{initial}} = M_1 v_{1x} \quad \text{or} \quad v_{2x}' = \frac{1}{2} v_{1x}.$$

c. Write an equation relating $v_{2y}'$ to other velocity components like $v_{1x}$ and $v_{1y}$.

$$P_{\text{final}} = M_1 v_{1y}' + 2M_1 v_{2y}' = P_{\text{initial}} = 0 \quad \text{or} \quad v_{2y}' = -\frac{1}{2} v_{1y}.$$

You now have two linear relations coupling the three unknowns $v_{1x}$, $v_{2x}'$, and $v_{2y}'$.

d. Now suppose the collision is "elastic" which means that the total kinetic energy is unchanged by the collision. Write the equation for this (which will give you a third equation, this time not linear, coupling the three unknowns).

$$\frac{1}{2} M (v_{1x})^2 + \frac{1}{2} (2M_1 v_{2x}')^2 + \frac{1}{2} (2M_1 v_{2y}')^2 = \frac{1}{2} M v_{1x}^2 + M v_{2x}'^2 + M v_{2y}'^2.$$

e. Compute the unknown $v_{1y}'$.

Substituting $v_{2x}' = \frac{1}{2} v_{1x}$ and $v_{2y}' = -\frac{1}{2} v_{1y}$ we get

$$\frac{1}{2} M v_{1x}^2 = \frac{1}{4} M v_{1x}^2 + \frac{3}{4} M (v_{1y}')^2 \quad \Rightarrow \quad (v_{1y}')^2 = \frac{1}{3} v_{1x}^2.$$

Thus $v_{1y}' = \frac{v_{1x}}{\sqrt{3}} = \frac{2.5\ \text{m/s}}{\sqrt{3}} \approx 1.4\ \text{m/s}$. 