PHYSICS 133 SPRING 2006 EXPERIMENT 7
SIMPLE HARMONIC MOTION

Introduction

In this lab, we study the phenomenon of simple harmonic motion for a mass-and-spring system and for a variety of pendulums. In a linear mass-spring system, the physical basis for this kind of motion is that the restoring force F exerted on a mass m that has been displaced from equilibrium must be proportional to $-x$, where x is the displacement. This relationship may be written

$$F = -kx \tag{1}$$

where k is a constant that characterizes the stiffness of the spring. A large value of k would indicate that the spring is difficult to stretch or compress. In the case of a simple pendulum, there is no spring, and k is replaced by the quantity (mg/L), where m is the mass of the pendulum bob, g is the acceleration due to gravity, L is the length of the pendulum, and x represents the (small) lateral displacement of the bob. Eq. (1) can be generalized to represent other physical situations. For example, the displacement might be given in terms of an angle, in which case the restoring variable would be a torque. See your textbook for examples.

Using Newton's second law, $F = md^2x/dt^2$, we can write Eq. (1) as a differential equation,

$$d^2x/dt^2 = -(k/m)x \tag{2}$$

This equation has as a possible solution the sinusoidal oscillation $x = A\cos\omega t$, which you can verify by direct substitution in Eq. (2). Here A is the amplitude and $\omega = \sqrt{(k/m)}$ is the circular frequency. The frequency depends on physical characteristics of the system. For example, $\omega = \sqrt{(k/m)}$ for a linear mass-spring system and $\omega = \sqrt{(g/L)}$ for small oscillations of a simple pendulum. Since the period $T = 2\pi/\omega$, we have $T = 2\pi\sqrt{(m/k)}$ for the mass-spring system and $T = 2\pi\sqrt{(L/g)}$ for the simple pendulum, respectively.

Equipment

- 1 air track with glider/spring oscillator,
- Low-friction pulley and thread,
- small masses,
- photogate,
- magnets,
- 1 simple pendulum,
- 1 meter stick pendulum, ring and disk pendulums
- 1 computer and interface box.

Method

Throughout this experiment, a system (glider between springs, assorted pendulums) will be displaced from equilibrium and the period, T, of oscillation will be measured. For the various oscillators, the dependence of the period on amplitude will be assessed. For true, simple harmonic motion, there should be no amplitude dependence.

Procedure

1. Measurement of the Spring Constant for the Mass-Spring Oscillator

Record the equilibrium position of the glider. Attach a piece of string to the glider and pass it over the low-friction pulley with a mass suspended from the free end of the string. Measure the displacement of the glider. Be sure that the string moves freely and does not scrape against anything. Repeat with 3 other masses for a total of 4 measurements. Graph the weight of the masses, mg, versus x, the displacement of the glider from equilibrium.

Q1. Is there a linear relationship between the weight and the displacement?

Measure the slope of the line fitted to these points on your graph. Determine the spring constant k from the slope of your plot.

2. Period of Glider/Spring System

Remove the string and place the photogate so that the "flag" atop the glider is near to, but does not obstruct the photogate when the glider is at equilibrium. Put the computer in PENDULUM MODE, which can be reached by way of MISCELLANEOUS TIMING MODES on the main menu screen.

a. Measure the period of oscillation for a small displacement of the glider. Record the mass of the glider.
b. Repeat the measurement of the period of oscillation after taping one and then two heavy masses to the glider. Does the measured $T = 2\pi\sqrt{(m/k)}$ for each mass as theory predicts?
c. Remove the masses and measure the period for several different amplitudes. Does the period change? How does this relate to the theoretical prediction?

3. Damped Oscillation

One air track has a glider with magnets attached so that there will be magnetic damping. By recording in MOTION TIMER mode, we can observe the maximum velocity as a function of time. Make a sketch of the plot in your lab notebook and discuss.

4. Simple Pendulum

Measure the length of the pendulum, L, and record the period of oscillation for different amplitudes.

Q2. Does $T = 2\pi\sqrt{(L/g)}$ for all amplitudes? Why or why not? How does this differ from the glider/spring oscillator?

5. Physical Pendulum

Measure the period of oscillation of a meter stick suspended at one end for small amplitudes.

Q3. Does $T = 2\sqrt{(2I/mgL)}$, where I is the rotational inertia of the meter stick about one end? Derive this relation.

6. (Optional) Disk Pendulum

Repeat part (5) for this pendulum. Measure T and derive and test the theoretical value for T.