Physics 472 Fall 2011 Problem set # 4 due Friday Oct. 7

1. Kittel p.128 problem 1

1. *Singularity in density of states.* (a) From the dispersion relation derived in Chapter 4 for a monatomic linear lattice of N atoms with nearest-neighbor interactions, show that the density of modes is

$$D(\omega) = \frac{2N}{\pi} \cdot \frac{1}{(\omega_m^2 - \omega^2)^{1/2}} \ .$$

where ω_m is the maximum frequency. (b) Suppose that an optical phonon branch has the form $\omega(K) = \omega_0 - AK^2$, near $K = 0$ in three dimensions. Show that $D(\omega) = (L/2\pi)^3(2\pi/A^{3/2})(\omega_0 - \omega)^{1/2}$ for $\omega < \omega_0$ and $D(\omega) = 0$ for $\omega > \omega_0$. Here the density of modes is discontinuous.

2. Kittel p.129 problem 5

5. *Grüneisen constant.* (a) Show that the free energy of a phonon mode of frequency ω is $k_BT \ln [2 \sinh (\hbar\omega/2k_BT)]$. It is necessary to retain the zero-point energy $\frac{1}{2}\hbar\omega$ to obtain this result. (b) If Δ is the fractional volume change, then the free energy of the crystal may be written as

$$F(\Delta, T) = \tfrac{1}{2}B\Delta^2 + k_BT \sum \ln [2 \sinh (\hbar\omega_{\mathbf{K}}/2k_BT)] \ ,$$

where B is the bulk modulus. Assume that the volume dependence of $\omega_{\mathbf{K}}$ is $\delta\omega/\omega = -\gamma\Delta$, where γ is known as the Grüneisen constant. If γ is taken as independent of the mode $\mathbf{K}$, show that F is a minimum with respect to Δ when $B\Delta = \gamma\Sigma\frac{1}{2}\hbar\omega \coth (\hbar\omega/2k_BT)$, and show that this may be written in terms of the thermal energy density as $\Delta = \gamma U(T)/B$. (c) Show that on the Debye model $\gamma = -\partial \ln \theta/\partial \ln V$. Note: Many approximations are involved in this theory: the result (a) is valid only if ω is independent of temperature; γ may be quite different for different modes.

3. Kittel, p.157, problem 1

1. *Kinetic energy of electron gas.* Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is

$$U_0 = \tfrac{3}{5}N\epsilon_F \ . \qquad (60)$$

4. Kittel, p. 158 problem 5

5. *Liquid He³.* The atom He^3 has spin $\frac{1}{2}$ and is a fermion. The density of liquid He^3 is 0.081 g cm^{-3} near absolute zero. Calculate the Fermi energy ϵ_F and the Fermi temperature T_F.