

Formulas that might be memorized (but it is equally if not more important to understand the derivation well enough that you could derive these in a minute or two.)

I. Math: the Brillouin zone.

$$\sum_{\vec{k}} g(\vec{k}) = \left(\frac{L}{2\pi}\right)^d \int d^d k g(\vec{k}) \text{ (in } d \text{ dimensions, neglecting spin degeneracy)}$$

$$N(\varepsilon) = \sum_{\vec{k}} \delta(\varepsilon_{\vec{k}} - \varepsilon)$$

$$\sum_{\vec{k}} g(\varepsilon_{\vec{k}}) = \int d\varepsilon N(\varepsilon) g(\varepsilon)$$

II. Free electrons: $\varepsilon(\vec{k}) = \hbar^2 k^2 / 2m$, $\psi_{\vec{k}} = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$

$$k_F = (3\pi^2 n)^{1/3}$$

$$N(\varepsilon_F) = 3n / 2\varepsilon_F \text{ (3d)}$$

$$N(\varepsilon) = N(\varepsilon_F) (\varepsilon / \varepsilon_F)^{d/2-1}$$

III. Debye model phonons: $\omega = vQ$, $\omega_D = vQ_D$

$$N(\omega) = C(\omega / \omega_D)^{d-1}, \text{ find } C \text{ by integration: } \int_0^{\omega_D} d\omega N(\omega) = d \text{ or other choice.}$$

IV. Statistical mechanics: $E = \sum_Q \hbar\omega_Q (n_Q + 1/2)$ etc. – partition functions, free energy, other thermodynamic relations. Specific heat.

$$n(Q) = \left(e^{\hbar\omega_Q / k_B T} - 1 \right)^{-1}$$

$$f(k) = \left(e^{(\varepsilon_k - \mu) / k_B T} + 1 \right)^{-1}$$

V. Transport $\vec{J} = -\kappa \vec{\nabla} T$ (Fourier's law) $\vec{j} = \sigma E$ (Ohm's law)

$$\vec{J} = \frac{1}{V} \sum_Q v_Q \varepsilon_Q N_Q \rightarrow \frac{1}{3} C \vec{v} \ell; \quad \hbar v_Q = \partial \omega_Q / \partial Q \qquad \vec{j} = \frac{-e}{V} \sum_{\vec{k}} v_k F_k; \quad \hbar v_k = \partial \varepsilon_k / \partial k$$

F and N are non-equilibrium distributions that relax back to f and n (Fermi-Dirac and Bose-Einstein). The equation of motion that describes this evolution is the Boltzmann equation, which assumes gas-like behavior or quasiparticles.