1. (5 points)

$$\ddot{x} + \omega_0^2 x = \varepsilon \sum_{n=0}^{\infty} (-1)^n \delta(t - nT/2)$$

Integrating both sides over a small time interval around the rest position:

($\int x dt$ vanishes for small variation in position)

$$\int_{\frac{nT}{2}-\epsilon}^{\frac{nT}{2}+\epsilon} (\ddot{x} + \omega_0^2 x) dt = \varepsilon \sum_{n=0}^{\infty} \int_{\frac{nT}{2}-\epsilon}^{\frac{nT}{2}+\epsilon} (-1)^n \delta(t - nT/2) dt$$

$$\mathrm{v}\left(\frac{nT}{2} + \epsilon\right) - \mathrm{v}\left(\frac{nT}{2} - \epsilon\right) = \varepsilon(-1)^n$$

Every time the oscillator passes through the rest position, its velocity changes by the amount of $\varepsilon$, always in the direction of the instantaneous motion. Therefore,

$$\mathrm{v}(nT^+) - \mathrm{v}(0) = (2n+1)\varepsilon$$

It is not parametric resonance since the amplitude increases linearly with time, not exponentially. This is ordinary resonance of an undamped oscillator.

2a. (10 points)

$$V'(r) = \varepsilon\sigma^{-1}\left[-12\left(\frac{\sigma}{r}\right)^{13} + 12\left(\frac{\sigma}{r}\right)^{7}\right] = 0$$

$$r = \sigma$$

$$V''(r) = 12\varepsilon\sigma^{-2}\left[13\left(\frac{\sigma}{r}\right)^{14} - 7\left(\frac{\sigma}{r}\right)^{8}\right]$$

$$k = 72\varepsilon\sigma^{-2} \quad \mu = m_{Ar}/2$$

$$\omega_0 = \sqrt{\frac{k}{\mu}} = 5.58 \times 10^{12}\ rad/s$$

$$\omega_0 = 0.89\ THz = 1.86 \times 10^4\ m^{-1} = 3.7\ meV = 42.6\ K$$

2b. (5 points)

$$V'''(r) = 12\varepsilon\sigma^{-3}\left[-182\left(\frac{\sigma}{r}\right)^{15} + 56\left(\frac{\sigma}{r}\right)^{9}\right]$$

$$V''''(r) = 12\varepsilon\sigma^{-4}\left[2730\left(\frac{\sigma}{r}\right)^{16} - 504\left(\frac{\sigma}{r}\right)^{10}\right]$$

$$V = \varepsilon[-1 + \frac{1}{2}72\sigma^{-2}(\mathrm{r} - \sigma)^2 - \frac{1}{3}756\sigma^{-3}(\mathrm{r} - \sigma)^3$$

$$+ \frac{1}{4}4452\sigma^{-4}(\mathrm{r} - \sigma)^4]$$

$$\alpha = -756\frac{\varepsilon}{\mu\sigma^3}$$

$$\beta = 4452\frac{\varepsilon}{\mu\sigma^4}$$

$$x^{(2)} = -\frac{\alpha}{2\omega_0^2}a^2 = 0.041\ \mathrm{nm}$$

$$\omega^{(2)} = \left(\frac{3\beta}{8\omega_0} - \frac{5\alpha^2}{12\omega_0^3}\right)a^2 = -2.9 \times 10^{12}\ rad/s$$

$$\omega^{(2)}/\omega_0 \sim 0.5$$

Shift of resonance frequency due to anharmonic effect is considerable, because argon pairs are only weakly bonded.

3. (5 points)

$$I = \begin{bmatrix} \frac{2}{3}Ma^2 & -\frac{1}{4}Ma^2 & -\frac{1}{4}Ma^2 \\ -\frac{1}{4}Ma^2 & \frac{2}{3}Ma^2 & -\frac{1}{4}Ma^2 \\ -\frac{1}{4}Ma^2 & -\frac{1}{4}Ma^2 & \frac{2}{3}Ma^2 \end{bmatrix}$$

4. (5 points)

$$I = \begin{bmatrix} \frac{2}{5}MR^2 & 0 & 0 \\ 0 & \frac{2}{5}MR^2 & 0 \\ 0 & 0 & \frac{2}{5}MR^2 \end{bmatrix} - \begin{bmatrix} \frac{2}{5}mr^2 + ma^2 & 0 & 0 \\ 0 & \frac{2}{5}mr^2 + ma^2 & 0 \\ 0 & 0 & \frac{2}{5}mr^2 \end{bmatrix}$$