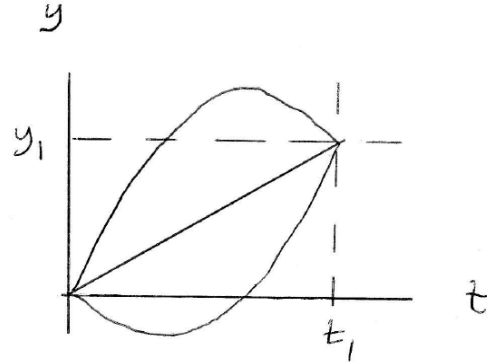


1. Action for 1d motion under gravity. The potential energy is $V(y) = mgy$, for a particle of mass m . Consider the family of one-dimensional motions from $(y=0$ at $t=0)$ to $(y=y_1$ at $t=t_1)$, with constant acceleration. One such “path”, with zero acceleration, is $y(t) = v_0 t$, where $v_0 = y_1/t_1$. A convenient parameterization of other paths is $y(t) = (v_0 + v_1)t - (2v_1/t_1)t^2/2$, where v_1 is an arbitrary number to parameterize different paths (see the figure).



(a) Find the part S_K of the action coming from kinetic energy alone. What value of v_1 minimizes it?

(b) Find the full action $S = S_K + S_V$. What value of v_1 minimizes it?

(c) What happens if you add a term $\Delta y(t) = A \sin(n\pi t/t_1)$ for some arbitrary integer n ? The “path” now has two free parameters, v_1 and A .

2. Time of descent in a harmonic potential.

(a) For a harmonic oscillator ($V = 1/2 kx^2$), a particle of mass m completes a cycle in time $T=2\pi/\omega$, and $\omega^2=k/m$. How long does it take to “fall” from rest at point b (assumed positive) to point a (assumed much smaller and positive)?

(b) The pendulum ($V = -mg\ell \cos\phi$, ϕ being measured from the resting position at the bottom of the circle) has a point of unstable equilibrium at $\phi = \pi$. Near that point, the potential is harmonic with a negative spring constant. How long does it take to “fall” from rest at angle α (extremely close to π , but slightly less) to angle β (very close to π , but much farther than α)? An alternate notation: let $x = \ell(\pi - \phi)$ ($\pi - \phi$ is assumed small) be the approximate horizontal displacement of the pendulum from its unstable resting point. Then the question is, how long does it take to go from a to b , where $a = \ell(\pi - \alpha)$ and $b = \ell(\pi - \beta)$.

(c) It is evident that these two problems are intimately related via a connection in the complex plane. As k changes sign, and ω changes from real to imaginary, the answer seems to have something to do with $\log(k)$. Why? I do not expect an elegant answer, because I didn’t find it myself, but I am sure an elegant solution exists that solves both these problems at once, and involves complex numbers and logarithms. A one-sentence answer will be enough.