Physics 501 – Classical Mechanics – Fall 2012   HW #2 due Friday Sept. 14

1. **Equation of a cycloid**. As done in lecture, use coordinates (x,y) for the horizontal and vertical coordinates of the position vector of a point on the rolling wheel, and use (0,0) as the starting point. For small (x,y) show that the cycloid obeys the approximate formula $y = C\,x^{2/3}$, and find the value of the coefficient C.

2. **Velocity on the optimum Bernoulli path**. This was not part of the challenge problem, so perhaps it was not known to Bernoulli, or perhaps he thought it might somehow simplify the challenge to ask about time-evolution.

(a) Show that the time evolution (of the particle of mass m falling on the inverted cycloid) corresponds to the point on the wheel moving at constant angular velocity $\dot{\phi}$, and find the value of this constant. **Note** – in case this seems counter-intuitive (as it did to me, at first), recall that the point where the wheel touches the ground is instantaneously stationary, even though $\dot{\phi} \neq 0$. Also, note that this is not just for small time, but for the full evolution, up to a possible moment where the particle departs from the cycloid and continues horizontally (below the starting point by $2r$) with constant velocity.

(b) Use this to show that the horizontal velocity on the optimal Bernoulli path increases quadratically in time, for small t, and find the coefficient.

3. "**Optimum" path** with least area and mean slope. Consider paths from $(x_0,y_0) = (0,0)$ to $(x_1,y_1)=(a,1)$. Find the path that minimizes $\boldsymbol{F}[y(x)] = \int_{x_0}^{x_1} dx\, f(y(x),y'(x),x)$, where the function $f(y,y',x) = y^2 + a^2 y'^2$. **Do this two ways**:

(a) Use the Euler-Lagrange equation directly, getting a second order differential equation.

(b) As done in class, show that, in place of the Euler-Lagrange equation, you can use instead a "first integral" of this equation, namely $f - y'\partial f/\partial y' = \text{const}$. This works only because $f(y,y',x)$ does not depend explicitly on the independent variable x. However, in this case, it doesn't simplify the algebra! The first procedure gives a linear 2nd-order equation which is not hard to solve, whereas the second gives a non-linear 1st –order equation which is not any easier.

4. Flexible pendulum. A spring has length L, spring constant K, and negligible mass. It has one end fixed at the point $(x,y) = (0,0)$. With no mass attached, its other end rests at $(x,y) = (0,-L)$. A point mass m is attached at the lower end, and motion under gravity in the (x,y) plane is observed. Use the usual circular coordinates $\boldsymbol{r} = (x,y) = r\,(\sin\phi, -\cos\phi)$ for the position of the mass. Unlike actual light-weight springs, this one does not flex, it can only stretch.

(a) Write the Lagrangian.
(b) Write the equations of motion for r and ϕ.
(c) Note that there is a complicated force of constraint at the fixed point of the spring. Why, or under what circumstances, is it appropriate not to include that in the problem?