Physics 501 – Classical Mechanics – Fall 2012   HW #3 due Friday Sept. 21

1. **Charged particle in a magnetic field**. As stated in class, the Lagrangian is $L = \frac{1}{2} m\vec{v}\cdot\vec{v} + q\vec{A}\cdot\vec{v}$ when there is no E-field present.

(a) For a homogeneous magnetic field, $\vec{B} = (0,0,B)$, using Cartesian coordinates and $\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$, write out the Lagrangian explicitly, and show that the correct Newtonian equations of motion are obtained.

(b) Show explicitly what happens to the equations of motion when you add an extra piece to the vector potential $\Delta\vec{A} = \vec{\nabla}\phi(\vec{r})$.

(c) What is the radius of the path followed by a proton with velocity 10^4 m/s moving perpendicular to a uniform field B = 1T?

(d) Find the form of the Lagrangian in cylindrical coordinates (ρ,ϕ,z).

(e) The energy is $\sum_i \dot{q}_i \partial L/\partial\dot{q}_i - L$. Construct the energy in Cartesian and in cylindrical coordinates. Comment on its form.

(f) The "canonical momentum" is $p_i = \partial L/\partial\dot{q}_i$. Express the energy in terms of the variables (p_x,p_y,p_z) and also in terms of (p_ρ,p_ϕ,p_z).

(g) If $\partial L/\partial q_i = 0$ for some coordinate q_i, then the corresponding momentum p_i is conserved. Find conservation laws of this kind for the charged particle in the constant magnetic field.

2. Action! Charged particle in a magnetic field. Suppose there is no field. A positively charged particle of mass *m* and charge *q* moves from $\vec{r}_0 = (0,0,0)$ to $\vec{r}_1 = (0,0,D)$ in time *T*.

(a) What is the action $S = \int_{t_0}^{t_1} dt L(\vec{r},\vec{v})$ (on the straight path, where $t_0 = 0$ and $t_1 = T$)?

(b) Suppose $\vec{B} = (0,0,B)$ and $T = m/qB$. What other (not straight, but correct Newtonian) paths go from $\vec{r}_0 = (0,0,0)$ to $\vec{r}_1 = (0,0,D)$ in time *T*?

(c) What is the action for some of these other paths? What does the principle of least action tell us?

3. Mechanical Similarity. Charged particle in a magnetic field. Suppose not just distance and time, but other quantities are scaled. What can you learn from this "mechanical similarity" about the behavior of the charged particle in a magnetic field?