Physics 501 – Classical Mechanics – Fall 2012 HW #4 due Friday Sept. 28

1. **Potential deduced from period**. The Lagrangian of a particle in d=1 is $L=mv^2/2-U(x)$, where $U(-x)=U(x)$ and $U(0)=0$. $U(x)$ has a single minimum, at x=0.
(a) The period is $T(E)=T$, a constant, for all $E>0$. What is the potential? You may use the method of LL sec. 12.
(b) The period is proportional to $E^{1/2}$ for all $E>0$. What can you say from "mechanical similarity?
(c) Use the method of LL sec. 12 to check (b).

2. Planetary orbit. Pluto's orbit has a mean distance to the sun of 39.439 au, where 1 au (astronomical unit) is 1.495×10^{11} m (distance from earth to sun.) Here "mean distance" means, not time-averaged, but spatially averaged, $r(\phi)$ averaged over ϕ. The eccentricity of its orbit is 0.2502, and inclination to the "ecliptic" is 17^o. Both exceed the values for mercury, which has the largest values of the eight planets. Calculate in SI units, where G = 6.67×10^{-11} Nm^2/kg^2, M (solar mass) = 1.99×10^{30} kg, and m (Pluto's mass) = 3×10^{24} kg.
(a) Calculate the aphelion and perihelion distances.
(b) Calculate the total energy (kinetic plus gravitational PE) of the sun/Pluto system (*i.e.*, neglect all other masses.)
(c) Calculate the orbital angular momentum, and orbital period in seconds and years, of the sun/Pluto system.
(d) If there were no other masses except the sun and Pluto, what would be the ratio of kinetic energy of the sun to kinetic energy of Pluto, in the center-of-mass frame?