Physics 501 – Classical Mechanics – Fall 2012 HW #5 due Friday Oct. 5

Note: the midterm exam is Friday Oct. 12 (in class, 10 – 10:53 am). There will be no homework due that week. HW # 6 will be due Oct. 19.

1. **Scattering and refraction**. On p. 54, Landau & Lifshitz solve the scattering from a spherical attractive well of radius *a*. Consider instead, a spherical repulsive well. You can take it to be infinitely massive. A Newtonian particle of mass *m* scatters elastically. Outside the well, the potential is $U=0$, while inside the well, the potential is $U = U_0$, a positive number. The discontinuous potential means an infinite outward force of zero range, $\vec{F} = -\vec{\nabla}U = -\hat{r}\partial U/\partial r = \hat{r}U_0\delta(r-a)$. This is a problem to be solved by conservation laws. Direct integration is not possible until carefully defined.

(a) Suppose the particle is incident with kinetic energy $mv_\infty^2/2 > U_0$ and impact parameter $\rho < a$. It arrives at the surface of the spherical well at angle α relative to the polar axis passing through the incident particle's point of arrival. Upon entering the well, its path is bent to angle β relative to the same axis. Find the relation between the scattering angle θ and the angles α and β. Find the formula for the effective index of refraction $n = \sin\alpha/\sin\beta$. Note that $0<n<1$.

(b) Use the method of Landau & Lifshitz to get the formula for ρ^2 as a function of θ. From this, the cross-section $d\sigma/d\Omega$ is easily derived as a function of θ (but there is no need to carry this out.)

(c) Explain what happens to particles incident beyond a critical impact parameter (α greater than Brewster's angle).

(d) Sketch the scattering angle θ as a function of impact parameter ρ/a for the case $n = 1/\sqrt{2}$. Describe the singularities in this function.

2. Small oscillations. Particles of mass *m, M*, and *m* move in one dimension, connected in series by two springs of spring constant *K*. This is a model for the CO_2 molecule. What are the frequencies and the patterns (eigenvectors) of the normal modes of oscillation? One of the normal modes is implicated in the greenhouse effect that worries us because of increased CO_2 concentrations in the atmosphere. Which mode? Why just that one?