Physics 501 – Classical Mechanics – Fall 2012 HW #6 due Wednesday Oct. 24

1. A harmonic oscillator (resonant frequency ω_0) is at rest up to time zero. Then it is subjected to a periodic series of impulses

$$F(t) = (m\varepsilon)\sum_{n=0}^{\infty} (-1)^n \delta(t - nT/2)$$

where $T = 2\pi/\omega_0$ is the period of the oscillator. What is the velocity at time $t = nT^+$. just after the oscillator passes through the rest position. Is it exponentially increasing with time? Is this parametric resonance?

2. A common way to represent the "Born-Oppenheimer" two-body potential between a pair of atoms with closed shells, is the "Lennard-Jones" potential,

$$V(r) = \varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - 2\left(\frac{\sigma}{r}\right)^{6}\right]$$

where r is the magnitude of the separation of the atoms, and ε and σ adjusted to fit some kind of data.

(a) Sketch this potential, indicating on the graph in units of ε and σ, where the minimum is. Taylor expand around the minimum, and find the Spring constant in units of ε and σ,. For argon atoms, the rough values are ε = 120K and σ = 0.34 nm. Find the resonant frequency in radians per second, Teraherz (THz), wavenumbers, millivolts, and degrees Kelvin. (All except the first are commonly used for things like this.)

(b) Expand to third and fourth order. Suppose the amplitude a of the oscillation in harmonic approximation is such that the energy $\mu\omega^2 a^2/2$ is 100K. Using the classical formulas in Landau and Lifshitz, what is the shift of the inter-atomic spacing and the shift of the resonant frequency caused by the lowest order appropriate anharmonic effects.

3. A solid homogeneous cube occupies the region of space given in some reference frame by $0 < x; y; z < a$. Calculate the inertia tensor of the cube in this reference frame.

4. A solid homogeneous ball of the radius R has a spherical cavity of the radius r with the center at the distance a from the center of the ball. Choose a convenient reference frame and calculate the inertia tensor of this body. You can assume $r+a<R$.