Physics 501 – Classical Mechanics – Fall 2012 HW #7 due Friday Nov. 2

1. Eq. 35.1 of Landau and Lifshitz expresses the angular velocity components Ω_i in the body frame in terms of the Euler angles. Do the same for the angular velocity components $(\Omega_X,\Omega_Y,\Omega_Z)$ in the lab frame.

2. Define the earth to be a symmetric top, and the "north pole" to mean the symmetry axis ($\mathbf{x}_3$) of the earth. It is inclined by an angle θ relative to the angular momentum vector **L** of the earth. The angular velocity vector **Ω** is inclined by an angle χ relative to the angular momentum vector **L**. The deviation $(I_3-I_1)/I_1$ from spherical symmetry of the earth is 0.0033. The earth's angular velocity vector, projected onto the earth's surface, is about 5 m from the north pole. The earth's radius is about 6.37×10^6 m.

a. For free rotations of a symmetric top, show that $\dot{\phi}\sin\chi = \dot{\psi}\sin(\theta-\chi)$. The notation is the same as in the figure given on the web in http://felix.physics.sunysb.edu/~allen/501/Notes/L26.pdf. These notes may help.
b. How large is χ for the earth, and how far from the angular velocity vector is the angular momentum vector of the earth (both vectors considered just as points near the pole where they intersect the earth's surface.)

3. As discussed in class (and in the notes referenced above), and proved by LL (p.115), the free rotation of a symmetric top has a simple description in the rotating (body) frame. It consists of a constant angular velocity Ω_3 around the symmetry axis x_3, plus a uniform-in-time rotation of the perpendicular component, around the symmetry axis. In other words, the angular velocity vector in this frame has constant magnitude. The question arises, how general is this. Derive the following equation (the time derivative is in the rotating frame) which provides the answer:

$$\frac{1}{2}\frac{d}{dt}\vec{\Omega}\cdot\vec{\Omega} = \frac{\Omega_1\Omega_2\Omega_3}{I_1I_2I_3}\left[(I_1-I_2)I_3^2+(I_2-I_3)I_1^2+(I_3-I_1)I_2^2\right]$$

What does this say about the "rotator" (a stick)?

2.