1. Explain how the Foucault pendulum works, and how its period depends on latitude (angle $\theta$ measured between equator and north pole from the center of the earth.) You can assume small angle oscillation, but it that really necessary?

2. Suppose a particle of mass $m$ and charge $q$ has a Lagrangian $L = m\dot{\vec{v}}^2/2 + q\vec{v} \cdot \vec{A}(\vec{r}) - U(\vec{r})$. 
   (a) Find the canonical momentum $\vec{p}$ and the Hamiltonian $H(\vec{r}, \vec{p})$. 
   (b) Write Hamilton’s equations of motion. 
   (c) Show carefully how to get the correct Newtonian equation $m\ddot{\vec{v}}/dt = \vec{F}$ (here $\vec{A}(\vec{r})$ does not depend on time.) 
   (d) Suppose $\vec{A}(\vec{r}, t)$ does depend on time, and that $U(\vec{r}, t) = q\Phi(\vec{r}, t)$. Repeat part (c).

3. A bug crawls on a rotating turntable (its axis is perpendicular to the plane of the turntable. Gravity is also perpendicular. The bug’s coordinates are $(x_0, y_0)$ or $(r_0, \phi_0)$ in the lab frame, and $(x, y)$ or $(r, \phi)$ in coordinates of the frame rotating with the turntable, and origin at the axis. The polar angle $\phi$ changes relative to the lab version in some fashion described by $\phi = \phi_0 + \dot{\theta}(t)$. 
   (a) Explain why (or under what interpretation) it is indeed correct and appropriate to express the Lagrangian this way in mixed coordinates. 
   (b) Rewrite the Lagrangian in rotating coordinates; find the momenta canonically conjugate to $(r, \phi)$; show that $H_{\text{rotating}} = H_{\text{lab}} - \dot{\theta} p_\phi$. Does it make sense that $H_{\text{lab}}$ is the same as $H_{\text{rotating}}$ when $\dot{\theta} = 0$?

4. The “phase portrait” is a diagram of the possible phase paths in the $(p,q)$ phase space. Suppose we have a particle of mass $m$ moving in one dimension, for positive $x$, under the potential $U(x) = \varepsilon[(\sigma/x)^{12} - (\sigma/x)^{6}]$ (the “Lennard-Jones” potential. Sketch the phase portrait.