Physics 501 – Classical Mechanics – Fall 2012 HW #8 due Friday Nov. 16

1. Explain how the Foucault pendulum works, and how its period depends on latitude (angle θ measured between equator and north pole from the center of the earth.) You can assume small angle oscillation, but it that really necessary?

2. Suppose a particle of mass *m* and charge *q* has a Lagrangian $\boldsymbol{L} = m\vec{v}^2/2 + q\vec{v}\cdot\vec{A}(\vec{r}) - U(\vec{r})$. **(a)** Find the canonical momentum $\vec{p}$ and the Hamiltonian $\boldsymbol{H}(\vec{r},\vec{p})$. **(b)** Write Hamilton's equations of motion. **(c**) Show carefully how to get the correct Newtonian equation $md\vec{v}/dt = \vec{F}$ (here $\vec{A}(\vec{r})$ does not depend on time.) **(d)** Suppose $\vec{A}(\vec{r},t)$ does depend on time, and that $U(\vec{r},t) = q\Phi(\vec{r},t)$. Repeat part (c).

3. A bug crawls on a rotating turntable (its axis is perpendicular to the plane of the turntable. Gravity is also perpendicular. The bug's coordinates are (x_0,y_0) or (r_0,ϕ_0) in the lab frame, and (x,y) or (r,ϕ) in coordinates of the frame rotating with the turntable, and origin at the axis. The polar angle ϕ changes relative to the lab version in some fashion described by $\phi = \phi_0 + \theta(t)$. Treating the bug as a particle of mass *m*, an appropriate Lagrangian is $\boldsymbol{L} = m\vec{v}_0^2/2 - U(r,\phi)$ where the velocity is lab frame and the potential varies in space. **(a)** Explain why (or under what interpretation) it is indeed correct and appropriate to express the Lagrangian this way in mixed coordinates. **(b)** Rewrite the Lagrangian in rotating coordinates; find the momenta canonically conjugate to (r,ϕ); show that $H_{\text{rotating}} = H_{\text{lab}} - \dot{\theta}p_\phi$. Does it make sense that H_{lab} is the same as H_{rotating} when $\dot{\theta} = 0$?

4. The "phase portrait" is a diagram of the possible phase paths in the *(p,q)* phase space. Suppose we have a particle of mass *m* moving in one dimension, for positive *x*, under the potential $U(x)=\varepsilon[(\sigma/x)^{12} - (\sigma/x)^6]$ (the "Lennard-Jones" potential. Sketch the phase portrait.