Physics 501 – Classical Mechanics – Fall 2012 HW #9 due Monday Nov. 26

Four tricks for free fall over the Thanksgiving holiday.

1. Flow in phase space. For 1-d free fall, phase space is 2-dimensional. Derive the formula for p(q) and plot the phase portrait with p vertical, q horizontal.

2. Action, generating trajectory. Consider free fall in 2d ($L=m(v_x^2+v_y^2)/2 - mgy$). For the actual path from (x_1, y_1, t_1) to (x_2, y_2, t_2), evaluate the action $S(x_1, y_1, t_1; x_2, y_2, t_2)$. Note that the initial velocity must not appear explicitly; it is implicit in the values of the end point positions and times. Derive S, and show that this serves as an F_1 generating function that transforms from the coordinates of the initial point to those of the final point.

3. Solve free fall by canonical transform. The Hamiltonian for free fall in 1d is $H(pq)=p^2/2m + mgq$, q being of course the z coordinate. By canonical transformation, you can make this into $H'(P,Q)=P$, which is easy to solve for $P(t)$ and $Q(t)$. Of course, this isn't much use unless you can find $p(P,Q)$ and $q(P,Q)$, and thus get $p(t)$ and $q(t)$. There is enough information to get an $F_4(p,P)$ generating function. Find this function and show how to solve the problem in P,Q coordinates, and then transform back to p,q, getting the familiar trajectory.

4. Solve free fall by Hamilton-Jacobi method. The Hamiltonian for free fall in 2d is $H(p,q)=p_x^2/2m + p_y^2/2m + mgy$. The solution of the Hamilton-Jacobi equation may be separated into parts depending separately on x, y, and t. Specifically, the action can be written as $S_x + S_y + S_t + S_0$, where S_0 contains all the irrelevant overall constants, and $H(S_x,S_y,S_t)$ is the sum of three parts that add to zero. The S_x part contains α_x, the S_y part, α_y, and the S_t part, $-(\alpha_x + \alpha_y)$. Find the action, an S_2 generating function. Use it to find the trajectory (x,y) *versus* t.