Phy 501 L4 9/5/2012

Landau-Lifshitz Ch II p 13 Say

DURING the motion of a mechanical system, the $2s$ quantities q_i and $\dot{q}_i$ ($i = 1, 2, \dots, s$) which specify the state of the system vary with time. There exist, however, functions of these quantities whose values remain constant during the motion, and depend only on the initial conditions. Such functions are called *integrals of the motion.*

$$\boxed{\begin{minipage}{0.9\textwidth}\end{minipage}}$$

The number of independent integrals of the motion for a closed mechanical system with s degrees of freedom is $2s-1$. This is evident from the following simple arguments. The general solution of the equations of motion contains $2s$ arbitrary constants (see the discussion following equation (2.6)). Since the equations of motion for a closed system do not involve the time explicitly, the choice of the origin of time is entirely arbitrary, and one of the arbitrary constants in the solution of the equations can always be taken as an additive constant t_0 in the time. Eliminating $t+t_0$ from the $2s$ functions $q_i = q_i(t+t_0, C_1, C_2, \dots, C_{2s-1})$, $\dot{q}_i = \dot{q}_i(t+t_0, C_1, C_2, \dots, C_{2s-1})$, we can express the $2s-1$ arbitrary constants $C_1, C_2, \dots, C_{2s-1}$ as functions of q and $\dot{q}$, and these functions will be integrals of the motion.

Not all integrals of the motion, however, are of equal importance in mech-

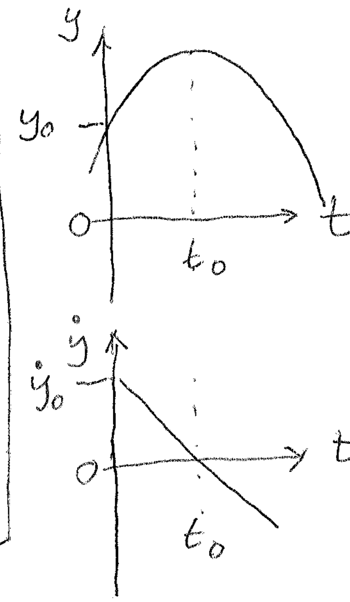

Illustration: 1d free fall

Solution
$$\left.\begin{aligned} y &= y_0 + \dot{y}_0 t - \tfrac{1}{2} g t^2 \\ \dot{y} &= \dot{y}_0 - g t \end{aligned}\right]$$
$2s = 2$ "arbitrary constants" — $y_0, \dot{y}_0$

Eliminate one of $(y_0, \dot{y}_0)$ in favor of t_0,

$0 = \dot{y}_0 - g t_0 \Rightarrow \dot{y}_0$ is replaceable by $g t_0$

$$\boxed{\dot{y} = -g(t - t_0)}$$

$$y = y_0 + \tfrac{1}{2} g t_0^2 - \tfrac{1}{2} g (t - t_0)^2 = \boxed{y_M - \tfrac{1}{2} g (t - t_0)^2 = y}$$

The solution is now written in terms of new "arbitrary constants" (t_0, y_M). Next, "eliminate $t - t_0$ from the $2s$ functions" yielding $2s - 1 = 1$ function not depending on $t - t_0$

Since $t - t_0 = -\dot{y}/g$, we have

$$y = y_M - \dot{y}^2/2g \quad \text{or} \quad \boxed{y_M = y + \dot{y}^2/2g}$$

Express the $2s - 1 = 1$ arbitrary constants (y_M) as functions of $(q, \dot{q}) = (y, \dot{y})$. Yes, done!

This is our $2s-1$ integrals of the motion $y_M = y + \dot{y}^2/2g$.