Physics 501 Monday Sept. 17, 2012

The 2-body central force problem has Lagrangian (1) $L = \frac{1}{2}\mu\,(\dot{r}^2 + r^2\dot{\phi}^2) - U(r)$. This is the **real** Lagrangian. The version (2) $L = \frac{1}{2}\mu\,\dot{r}^2 + \frac{M^2}{2\mu r^2} - U(r)$ is numerically correct, since $M = \mu r^2\dot{\phi}$ is conserved. [I switched from *L* to *M* as a symbol for angular momentum, since it is hard to find a script "*L*" to use for "Lagrangian."]

The time-independence of L means that (3) $E = \sum_i \dot{q}_i\,\partial L/\partial\dot{q}_i - L$ is conserved, where q_i are **all** the generalized coordinates. These are (r,ϕ). In class I was seduced by the appearance of version (2) as a Lagrangian with one degree of freedom. It really has two degrees of freedom, but since *M* is conserved, for many purposes you can forget the ϕ degree of freedom. But it is really there; ϕ evolves in time. So when you use (3) to calculate E, you have to include both partial derivatives of L, not just one, as I foolishly thought by looking at (2). The result, of course, is

$$E = \frac{1}{2}\mu\,(\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = \frac{1}{2}\mu\,\dot{r}^2 + \frac{M^2}{2\mu r^2} + U(r)$$