# Physics 540 Fall 2005

HW # 10 Due Tuesday November 22

1. Consider a system of two Ising spins obeying the Hamiltonian

$$H = -\mu B(\sigma_1 + \sigma_2) - J\sigma_1\sigma_2$$

They are in equilibrium at temperature T; the spins σ are 1 or -1; the exchange coupling J is positive.

a. Write the partition function Z.
b. Explain how to compute $<\sigma_1 + \sigma_2>$ and $<\sigma_1\sigma_2>$ from derivatives of logZ.
c. Evaluate $<\sigma_1 + \sigma_2>$ and $<\sigma_1\sigma_2>$.
d. Evaluate the susceptibility $\chi = (\partial M/\partial B)_{B=0}$. For small T (i.e. $k_BT << J$), this system can be called a "superparamagnet." Explain what this means, and discuss in what sense the system is magnetically ordered and in what sense it is not.

2. Consider a similar system but for two Heisenberg spins, but where spins σ are twice the s=1/2 angular momentum vectors, with the usual non-commuting properties of the Pauli matrices. The field B is also a vector, chosen in the z direction:

$$H = -\mu B\cdot(\sigma_1 + \sigma_2) - J\sigma_1\cdot\sigma_2$$

What are the eigenvalues of H? Repeat parts a, b, c from above.

3. For both Ising and Heisenberg systems as in 1 and 2, evaluate the leading high T expansion of log Z (Kittel eq. 23.20)

$$\log Z = \log \mathrm{tr}\, 1 - [\mathrm{tr}\, H/\mathrm{tr}\, 1]/k_BT + [\mathrm{tr}\, H^2/\mathrm{tr}\, 1 - (\mathrm{tr}\, H/\mathrm{tr} 1)^2]/2(k_BT)^2$$

and compare with problems 1a and 2a.

4. For the same system as in 2, explain why the J<0 problem (antiferromagnetic Heisenberg exchange coupling, with B=0) gives different physics from J>0, whereas for the Ising system of problem 1, the physics doesn't depend on the sign of J.