Physics 540 Fall 2005
Homework Assignment No. 12 due Tuesday Dec. 13

These are practise problems – don't hand them in. Solutions will be posted on the web, Wednesday Dec. 14

1. On p. 156 Kittel describes a density n of particles in one dimension which at time t=0 has the form $n(x,0) = N\delta(x)$. The particles are diffusing, so the density obeys

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial t^2}.$$

Derive Kittel's solution (31.20) and verify eq. (31.21). You can do this by writing

$$n(x,t) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} n(q,t) e^{iqx}$$

and deducing the form of $n(q,0)$, and solving for $n(q,t)$ and then $n(x,t)$. The Gaussian integral

$$\int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{ia(q-i\beta)^2}$$

has a value which is independent of the value of β [why?].

2. Random walk.

a. In one dimension, if the probability of a forward step is p and a backward step is q ($p + q = 1$), then show that after N steps, the probability of making n_1 steps forward (and $n_2 = N - n_1$ steps backward) is

$$W(n_1) = \frac{N!}{n_1!(N-n_1)!} p^{n_1} q^{N-n_1}.$$

b. When $p = q = 1/2$, find the variance $\bar{n_1^2}$, and show that the approximation

$$W(n_1) \approx \frac{1}{\sqrt{2\pi \bar{n_1^2}}} e^{-n_1^2/2\bar{n_1^2}}$$

should be good. In what sense should this be good?

c. What does this have to do with the central limit theorem?

d. Connect the random walk with the diffusion process in problem 1; how is the diffusion constant D related to the properties of the walk?