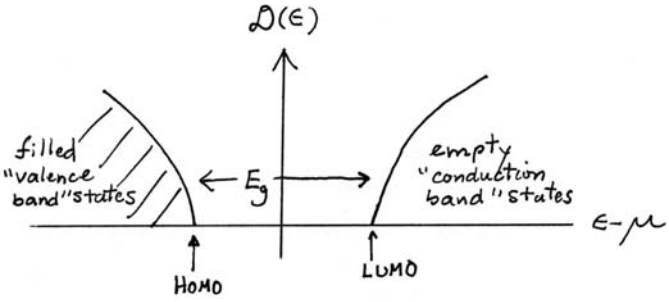
Physics 540 Fall 2005

HW # 7 Due Tuesday November 1

1. The chemical potential μ of the free Fermion gas is fixed by the requirement that the mean number of particles in the box is $N=\Sigma_i f_i$, and the Fermi-Dirac function is $(e^x+1)^{-1}$, with $x=(\varepsilon-\mu)/k_BT$. Find μ at high T. Explain why μ/k_BT should be negative and not small in this case. Show that the resulting formula for μ agrees with the result derived for the classical gas, except for a factor $(2s+1)$ representing spin degeneracy. Explain why this factor was (incorrectly) missed in the classical derivation.

2. Solid, crystalline silicon, when pure, has a HOMO-LUMO gap (E_g) of about 1eV. This means that the lowest unoccupied molecular orbital (LUMO) tends to remain unoccupied by electrons at $T < 300$K, and the highest occupied molecular orbital (HOMO) tends to remain filled by electrons (empty of holes). At higher T, there starts to be a noticeable population of thermally excited electrons and holes. If an extra electron is injected into silicon at low T, it goes into the "band" of empty levels starting at the LUMO level, with typical energy k_BT above the bottom of the band. In device applications, silicon is intentionally "doped" by substituting donor (or acceptor) atoms like As (or Al) in place of some of the Si atoms. It is normal to have doping levels in the range n = 10^{15} to 10^{19} donors per cm^3. In real life, the presence of foreign atoms alters the energy levels in interesting ways which we will neglect for the present purposes. Ignore the foreign atoms and put a density n of electrons into the empty levels, and treat them as a quantum ideal gas, non-relativistic, of particles whose mass is 0.3 times the mass of the electrons. You can choose the zero of energy at the LUMO level. At T=0 they make a degenerate Fermi system. (a) What is the fractional doping level x = # of dopants / # of Si atoms? (b) Compute the Fermi energy E_F. Do these for the two cases $n = 10^{15}$ and 10^{19}. Express E_F in degrees K. This model works quite well for understanding "heavily doped" semiconductors.



3. From the specific heat calculation $C_V=(\partial U/\partial T)_{N,V}= \gamma(N,V)T$, we can deduce that there is a temperature-dependent term $\gamma(N,V)T^2/2$ in the energy $U(N,V,T)$ of the orbital gas model of Fermions. This is, of course, smaller by k_BT/E_F than the energy $U=3Nk_BT/2$ of the classical monatomic gas. Unlike the classical gas, there is also a non-zero energy at T=0 which is not derivable from specific heat. You might guess that it would have something to do with the value of E_F. (a) Find the formula for the case of a "free" Fermion gas (non-relativistic particles in a box). (b) For Al metal with E_F=11eV, what is the numerical value of U/N and of pressure p at T=0? (express p in Pa and in atmospheres.) The answers are not "realistic" for actual Al metal; the interactions between particles shift these numbers, especially p.

4. The classical gas of particles with magnetic moments μ has a susceptibility $\chi=(\partial M/\partial H)_{T,V,N}$ equal to $N\mu^2/k_BT$. For the "degenerate" (meaning low T) orbital gas of Fermions, find the corresponding low T susceptibility (just the T=0 answer is fine.) You should express your answer in terms of the density of states $D(E_F)$. It is not necessary to assume a "free" gas (free particles in a box).