1 Cerium phase diagram:

We start from Clausius Clapeyron equation:

\[ \frac{dp}{dT} = \frac{s_\alpha - s_{\alpha'}}{v_\alpha - v_{\alpha'}} \]

for molecular volumes and entropies. Evaluate on the coexistence seam:

\[ \frac{dp}{dT} = -\frac{30 \text{ Kbar}}{1000 \text{ K}} \]

we find:

\[ s_\alpha - s_{\alpha'} = \left(-0.044 \times 2.65 \times 10^{-29} \frac{m^3}{\text{atom}}\right) \times \left(-\frac{30 \text{ Kbar}}{1000 \text{ K}}\right) = 3.5 \times 10^{-24} \frac{J}{K \text{ atom}} \]

the heat could be obtained from \( dQ = TdS \) and is:

\[ 400K \times \Delta s = 1.4 \frac{J}{\text{atom}} = 0.009 \frac{eV}{\text{atom}} \]

which should be absorbed on the transition \( \alpha \rightarrow \alpha' \).

2 \( d = 2 \) gas:

The one particle partition function is:

\[ z_1 = \frac{A}{\lambda^2} = \frac{2\pi mAkT}{h^2} \]

and the full partition function is therefore:

\[ Z(T,V,N) = \frac{1}{N!} z_1^N = \frac{1}{N!} \left(\frac{2\pi mAkT}{h^2}\right)^N \]

Following the advice we ignore Gibbs factor as we are not concerned with explicit \( N \)–derivatives of the partition function (though it DOES change the entropy!).
The free energy reads:

\[ F = -kT \log Z = -NkT \log \left( \frac{2\pi m AkT}{\hbar^2} \right) \]

The pressure is:

\[ p = -\frac{dF}{dA} = \frac{NkT}{A} \]

which is analogous to the ideal gas in \( d = 3 \).

The entropy is given by:

\[ S = -\frac{dF}{dT} = Nk \log \left( \frac{2\pi m AkT}{\hbar^2} \right) + Nk \]

The Energy could be calculated by:

\[ U = F + TS = NkT \]

which is what we expect from equipartition. This result IS independent of the question whether Gibbs factor is introduced in the calculation.

3 Magnetisation issues:

The one particle partition function is:

\[ z_1 = e^{-\beta E_0} + e^{-\beta(E_0+\Delta-\mu H)} + e^{-\beta E_0} + e^{-\beta(E_0+\Delta)} + e^{-\beta(E_0+\Delta+\mu H)} \]

The magnetisation would not be affected by Gibbs factor issues and we can write:

\[ Z_N = z_1^N \Rightarrow F = -NkT \log \left( e^{-\beta E_0} + e^{-\beta(E_0+\Delta-\mu H)} + e^{-\beta E_0} + e^{-\beta(E_0+\Delta)} + e^{-\beta(E_0+\Delta+\mu H)} \right) \]

or using the more convenient formulation:

\[ F = -NkT \log \left( e^{-\beta E_0} \left[ 1 + e^{\beta \Delta} + 2 \cosh(\beta \mu H) \right] \right) \]

the magnetisation is:

\[ M = -\frac{dF}{dH} = \frac{2N\mu \sinh(\beta \mu H)}{1 + e^{\beta \Delta} + 2 \cosh(\beta \mu H)} \]

the susceptibility is:

\[ \lim_{H \to 0} \frac{M}{H} = \frac{2N\mu}{3 + e^{\beta \Delta}} \lim_{H \to 0} \frac{\sinh(\beta \mu H)}{H} = \frac{2N\mu}{3 + e^{\beta \Delta}} \beta \mu \]