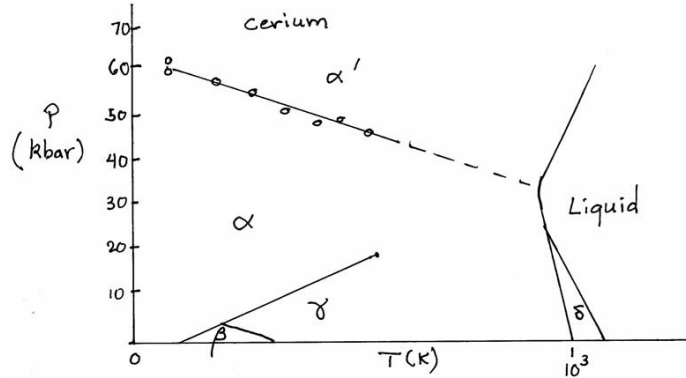


# Phy 540 Fall 2005 Midterm Exam Thursday Oct. 20

8:20-9:40 am room B-131

Closed book. Calculators allowed.

1. The picture shows the  $p$ - $T$  phase diagram (measured by King, *et al.*, Phys. Rev. B **1**, 1380 (1970)) of elemental cerium (atomic number 58, atomic mass  $M=140$  amu.) Except for the liquid region, all phases shown are solids. The vapor phase occurs at such low  $p$  that it is indistinguishable from the horizontal axis on this scale. Along the lines where two different phases coexist, the Gibbs energy  $G(p,T,N)$  per atom is the same in the two phases. It was measured that at  $(p,T)=(50 \text{ kbar}, 400\text{K})$ , the volume of the sample diminished by 4.4% (from the  $\alpha$  phase value  $V/N=2.65 \times 10^{-29} \text{ m}^3/\text{atom}$ ) on crossing the  $\alpha \rightarrow \alpha'$  phase boundary. How much (latent) heat was added (or removed)? (Note:  $1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ N/m}^2$ ; conventional physicist's units for latent heat  $L$  would be J/mole or eV/atom. I want a number accurate to 10%.)



2. A gas of  $N$  monatomic molecules of mass  $M$  is confined to a 2-dimensional box of area  $A$ . Each particle has kinetic energy  $(p_x^2 + p_y^2)/2m$ . Using the canonical ensemble, find the partition function  $Z(T,A,N)$  (classical version; you may insert the ‘‘Gibbs factor’’  $1/N!$  and normalizing factor  $h^{2N}$  if you wish; these won't change anything in this problem.) Use the partition function to find  $F(T,A,N)$  and use this to find  $p(T,A,N)$ ,  $S(T,A,N)$ , and  $U(T,A,N)$ . How do the answers differ from the familiar classical ideal gas formulas for  $p$  and  $U$ ? (Note: the 2-d pressure  $p=\Delta W/\Delta A$  has units N/m.)

3. Consider  $N$  atoms in volume  $V$ , each with a non-degenerate ground state of energy  $E_0$  and a triply degenerate excited state of energy  $E_0 + \Delta$ . The excited state energies are shifted by  $-\mu H$ ,  $0$ , and  $+\mu H$ , when the field  $H$  is applied and when the magnetic moment  $\mu$  points along, perpendicular to, or against the applied field. The ground state has no moment and its energy is not shifted by the field. Other degrees of freedom such as motion in space are decoupled from these 4 internal states and therefore give only multiplicative parts of the partition function  $Z$  which do not affect the magnetic part. Find the magnetization  $M=(n_\uparrow - n_\downarrow)\mu/V$  and the susceptibility  $\chi=M/H$  (in the  $H \rightarrow 0$  limit.) Does the result look more familiar if  $\Delta/k_B T$  is large and negative? Why?