1. Consider $N$ atoms of type A located at positions $x = a_n = 2nc$ ($n=1,2,...,N$), and an equal number of type B located at $x = b_n = (2n+1)c$. Each atom has one electron and one orbital, $\psi_A(x-a_m) = |mA>$, and $\psi_B(x-b_p) = |pB>$. Orbitals are all orthonormal, $<mA|nA> = \delta_{mn}$, $<mB|nB> = \delta_{mn}$, and $<mA|nB> = 0$. There are periodic boundary conditions on the wavefunctions $\psi(x+2Nc) = \psi(x)$. The Hamiltonian for each electron is $H = p^2/2m + V(x)$ where $V(x+2a) = V(x)$. The Hamiltonian matrix elements are $<mA|H|nA> = \delta_{mn}\epsilon_A$, $<mB|H|nB> = \delta_{mn}\epsilon_B$, and $<mA|H|nB> = 0$ unless $a_m$ and $b_n$ are nearest neighbors, in which case $<mA|H|nB> = -t$. Find the Bloch state eigenvalues $\epsilon_n(k)$, and sketch $\epsilon_n(k)/t$ for the case $t=1$ eV and $\epsilon_A - \epsilon_B = t$.

2. The graph below is from a neutron scattering experiment on solid crystalline (rare gas) krypton ($M_{Kr} = 83.8$ amu). The paper is J. Skalyo, Y. Endoh, and G. Shirane, Phys. Rev. B 9, 1797 (1974). Kr has fcc crystal structure, and lattice constant $a=5.7\text{Å}$.

- At what temperature $T$ are you approximately in the classical limit for heat capacity?
- What is the value of the classical heat capacity (in $\text{J/moleK}$ and in $\text{J/kgK}$)?
- The left-most panel ($\Gamma$ to X along $\Delta$) is the (100) direction, and the point “X” is the Brillouin zone boundary along (100) with $k = (2\pi/a)(1,0,0)$. What is the velocity of longitudinally-polarized sound (in m/s) in this direction?
- Why are only two branches shown in the left and right, while three are shown in the two middle panels?
- Are there “optical” phonons not shown here?
- Estimate the magnitude $\sqrt{<u^2>}$ of zero point vibration of Kr atoms in this crystal.
- Make an intelligent guess about the energy gap $E_g$ for electronic excitations in Kr.