
3.1.1. Given a group $G$ of $g$ elements $A_1, A_2, \ldots, A_g$ and a sufficiently arbitrary basis function $\Psi$ such that the set of $g$ basis functions $\Psi_i = A_i \Psi$ is an independent set, show that the $\Psi_i$ form a basis for a representation

$$R \Psi_j = \sum \Psi_i D_{ij}^{\text{reg}}(R)$$

known as the regular representation, whose elements are 1 or 0 with

$$D_{ij}^{\text{reg}}(R) = \delta(R, A_i A_i^{-1})$$

2. Lax p. 106.

3.1.2. (a) Show that for the regular representation

$$\chi^{\text{reg}}(E) = g, \quad \chi^{\text{reg}}(R) = 0 \quad \text{for} \quad R \neq E.$$

(b) Show that representation $\Gamma_i$ of dimension $l_i$ is contained $l_i$ times in the regular representation.

(c) Prove Burnside’s theorem, $\Sigma(l_i)^2 = g$ (Eq. 1.5.9).

3. Lax p. 131

4.3.2. Show that a measurement of electrical conductivity in a crystal of cubic symmetry is independent of the orientation of the crystal. Is this also true of optical absorption?

4. Lax p. 131

4.3.3. Show that the presence of a reflection plane containing the principal axis forces $c$ to equal 0 in Eq. 4.3.10.

Eq. 4.3.10 applies correctly to the group $C_3$, not $C_{3v}$, as this problem clarifies.

$$\begin{bmatrix} a & c & 0 \\ -c & a & 0 \\ 0 & 0 & b \end{bmatrix}$$

(4.3.10)