

“Phonons”

Vibrational normal modes

- Evolution from molecule to solid
- Heat conduction
- What are vibrations like in glasses?

P. B. Allen, visiting Applied Physics at Columbia
On leave from SUNY Stony Brook

Euken

Einstein

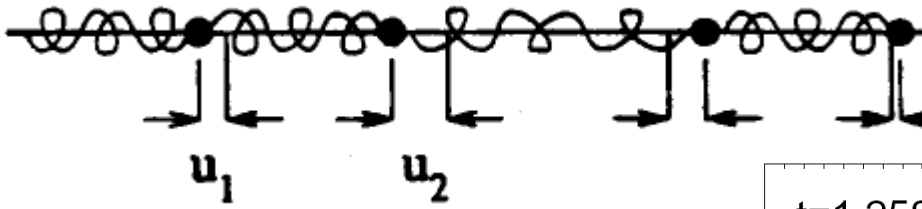
Debye

Born

Peierls

Frenkel

Brockhouse

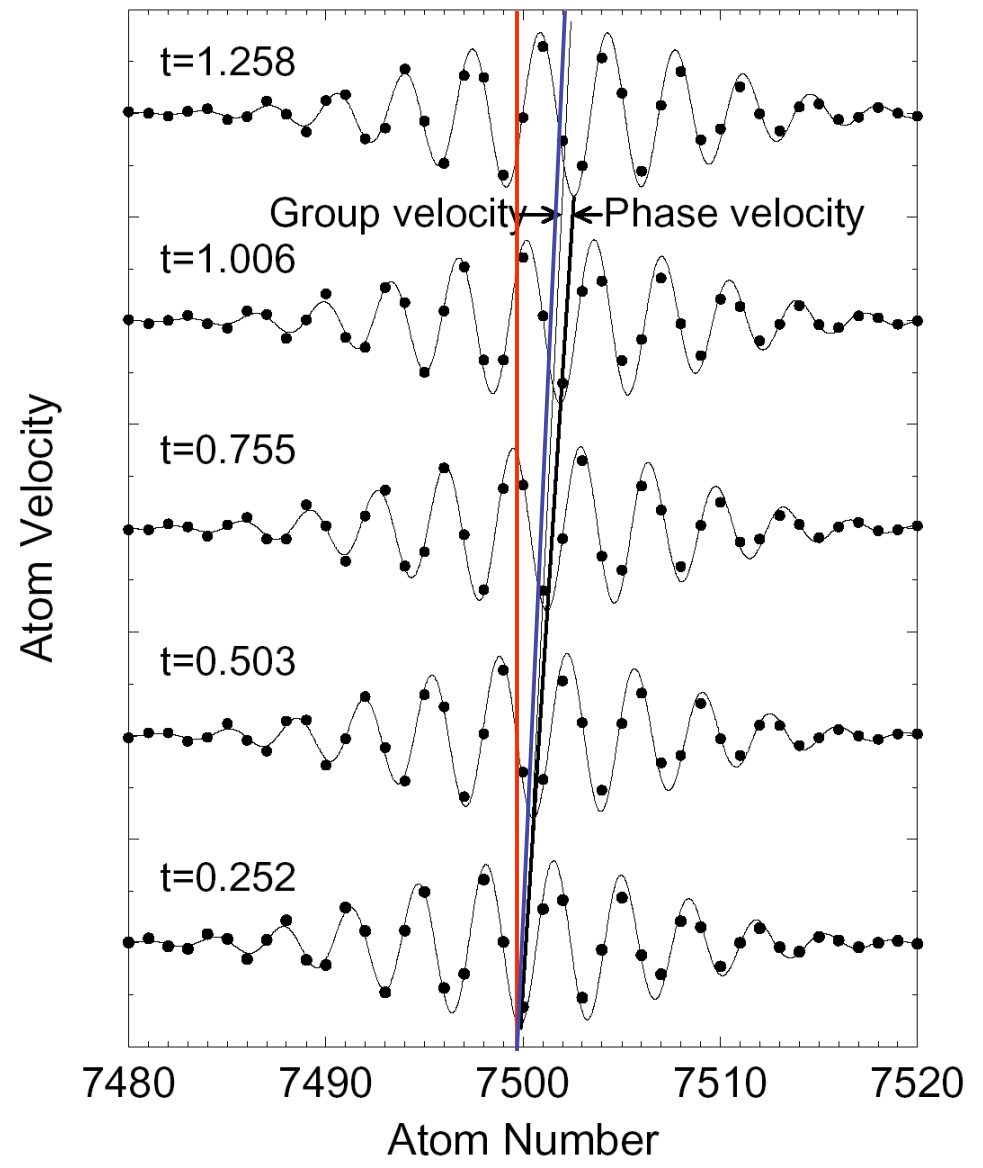
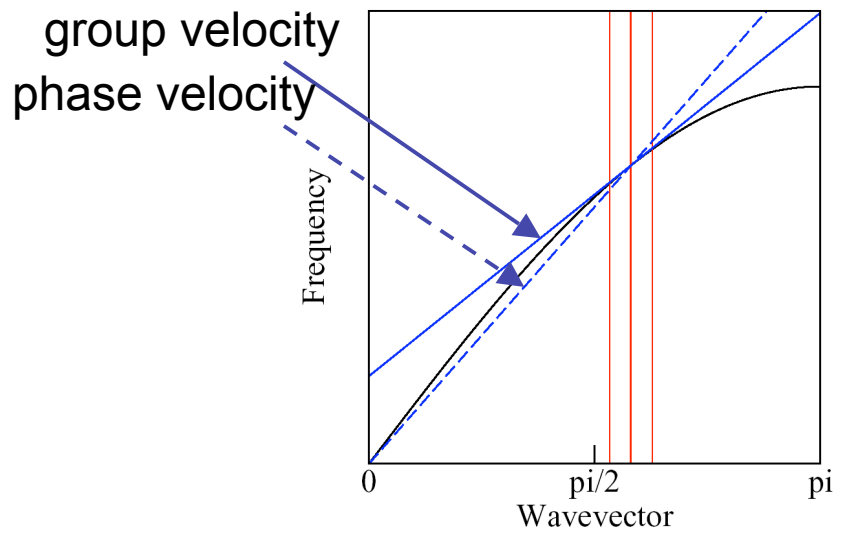


$$V = \frac{1}{2} \sum_{n=1} k_n (u_n - u_{n+1})^2$$

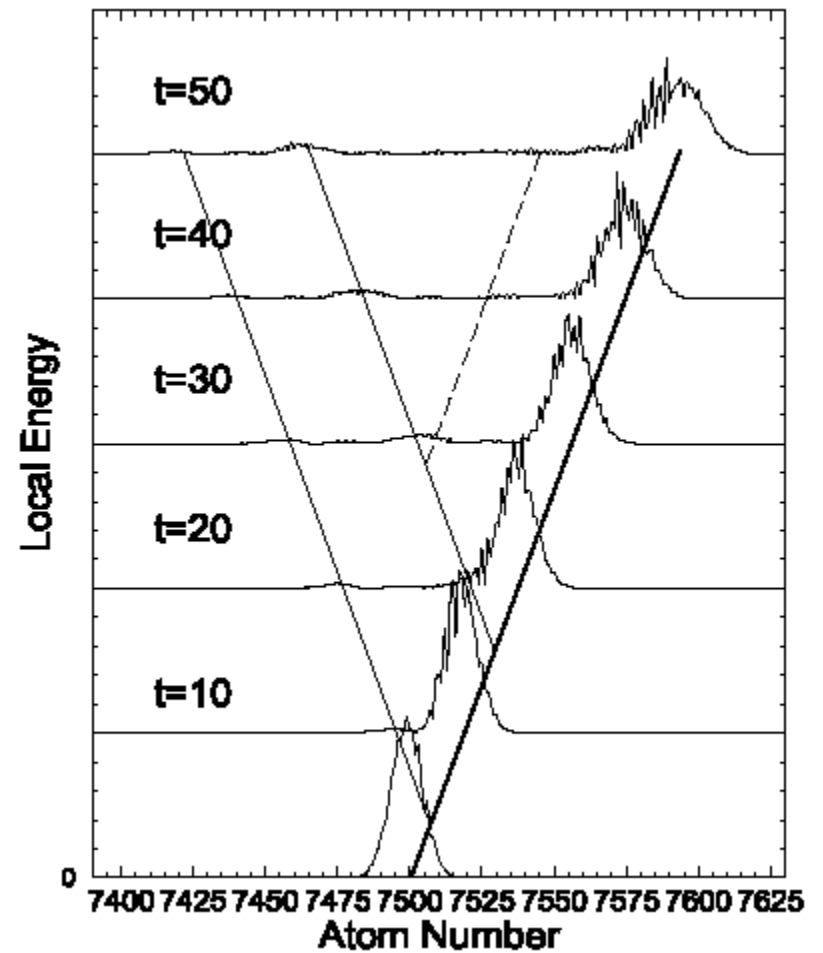
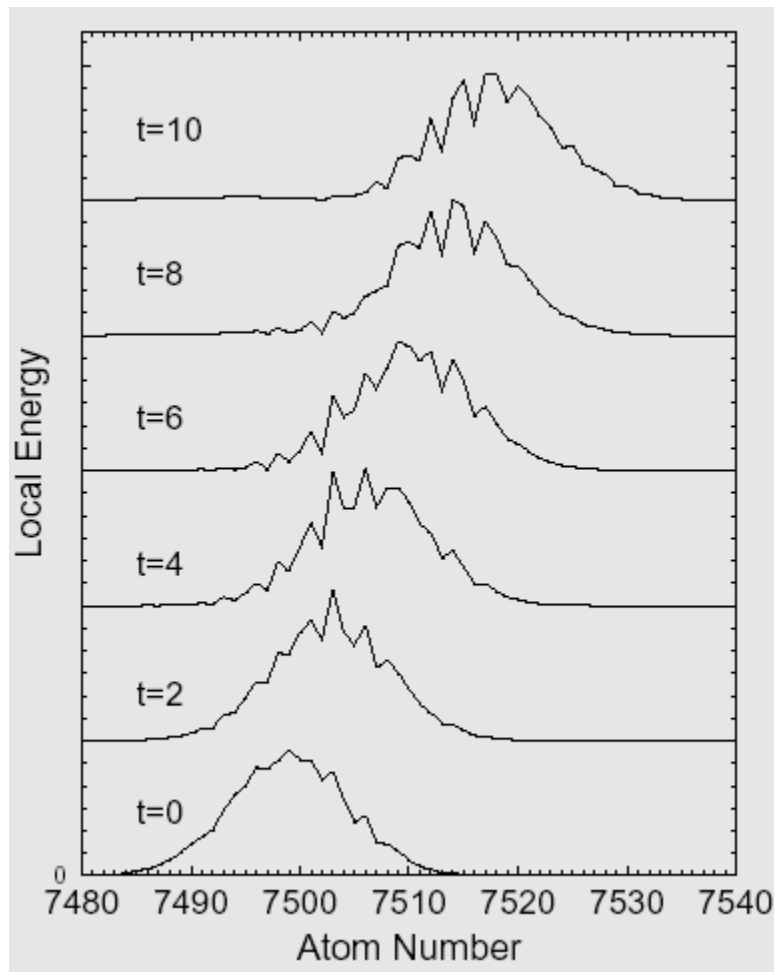
$$M_n \frac{d^2 u_n}{dt^2} = - \frac{\partial V}{\partial u_n}$$

$$u_n(t) = |A_Q| \cos(Qna \pm \omega_Q t + \phi_Q)$$

$$\omega_Q = 2\sqrt{k/M} \sin(Qa/2)$$

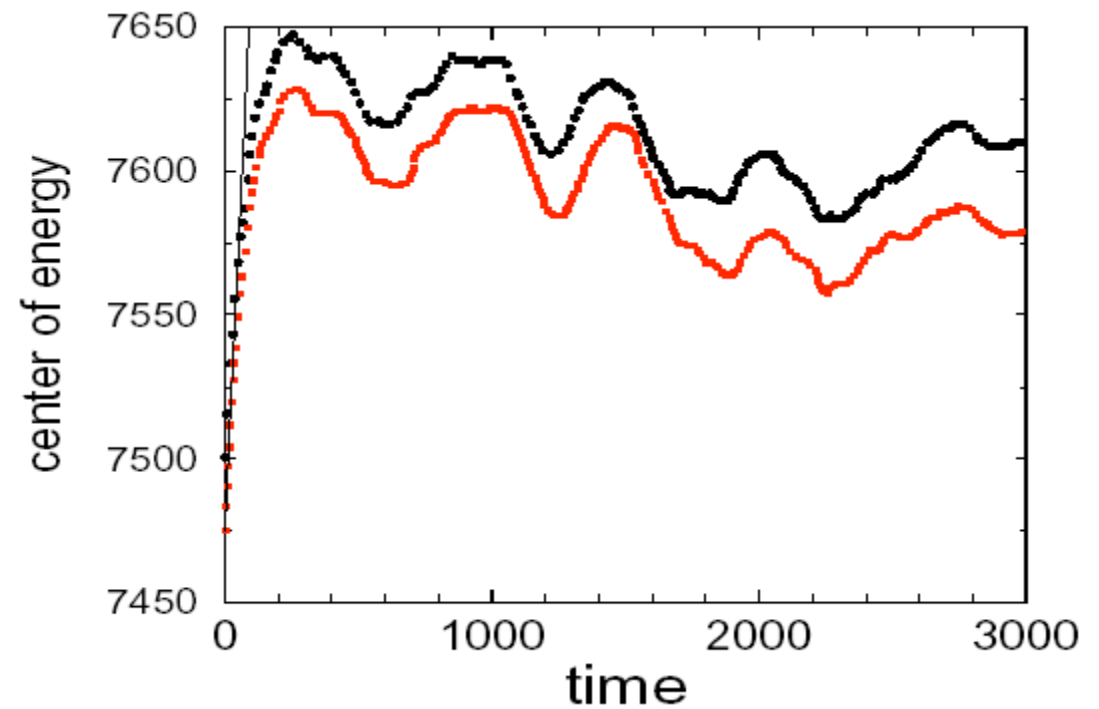
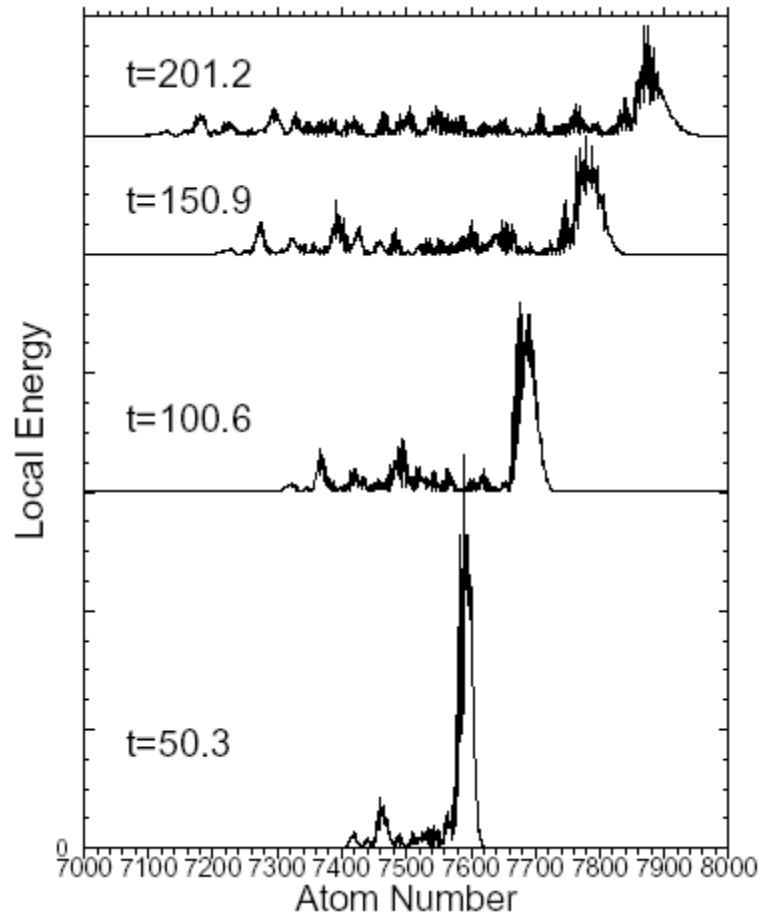


Energy transport, weakly disordered 1d chain
 $0.93 < K < 1.07$ uniform random deviations



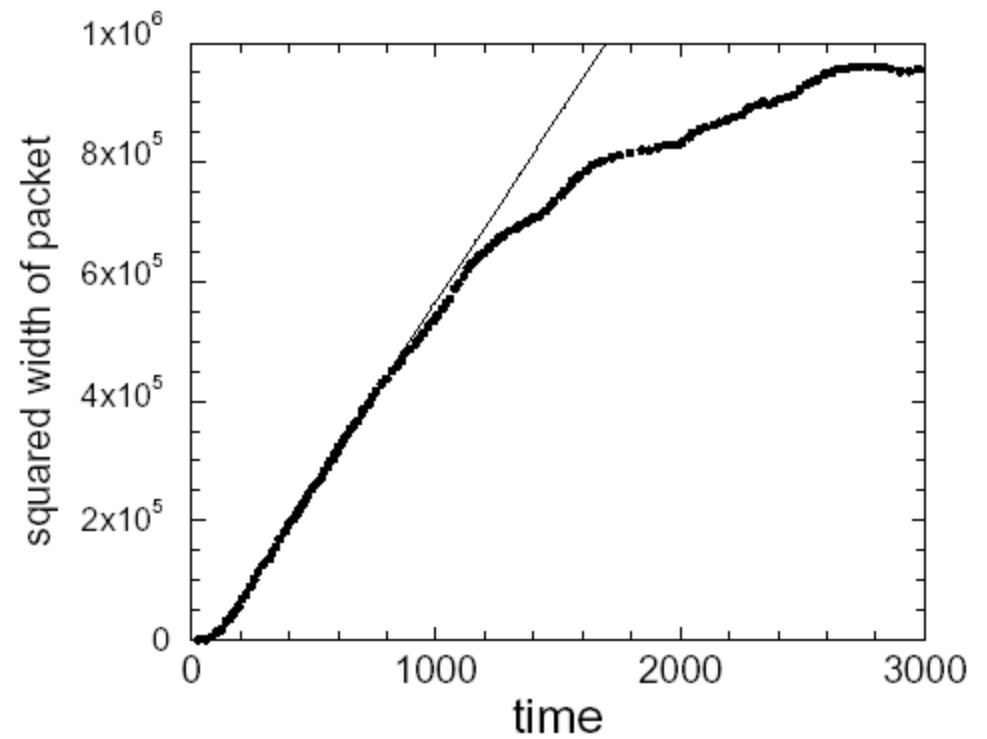
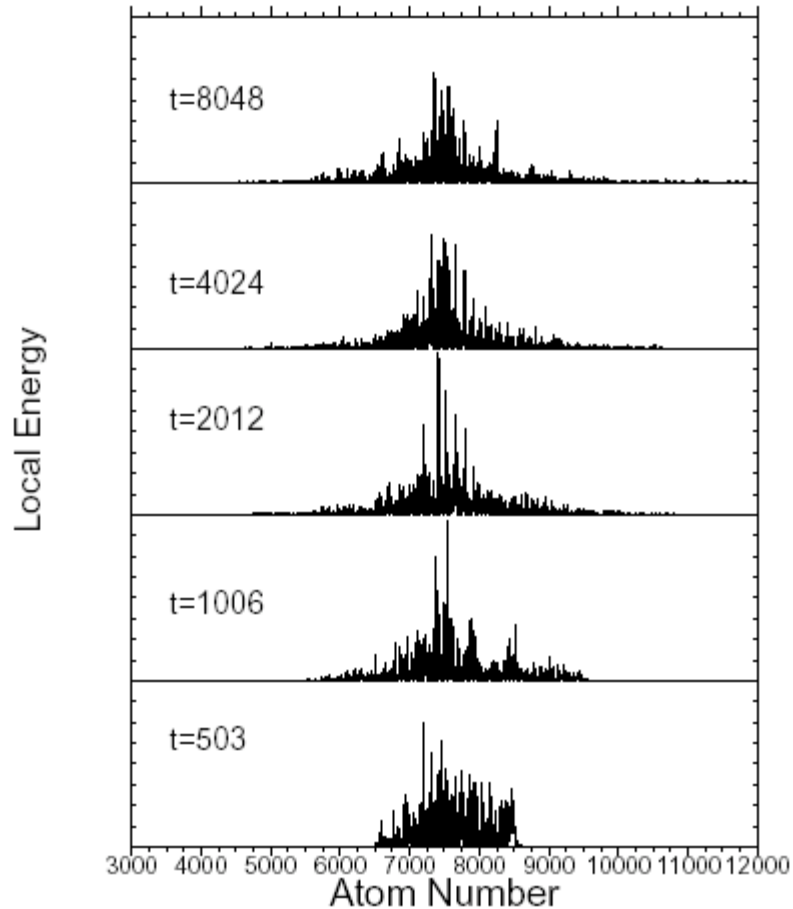
Ballistic motion of wavepacket for $0 < t < 100$
Center of energy stops systematic motion by $t=200$

Non-Chaotic Evolution



Diffusive motion, $100 < t < 1000$

Localization, $t > 2000$ (no further energy transport)



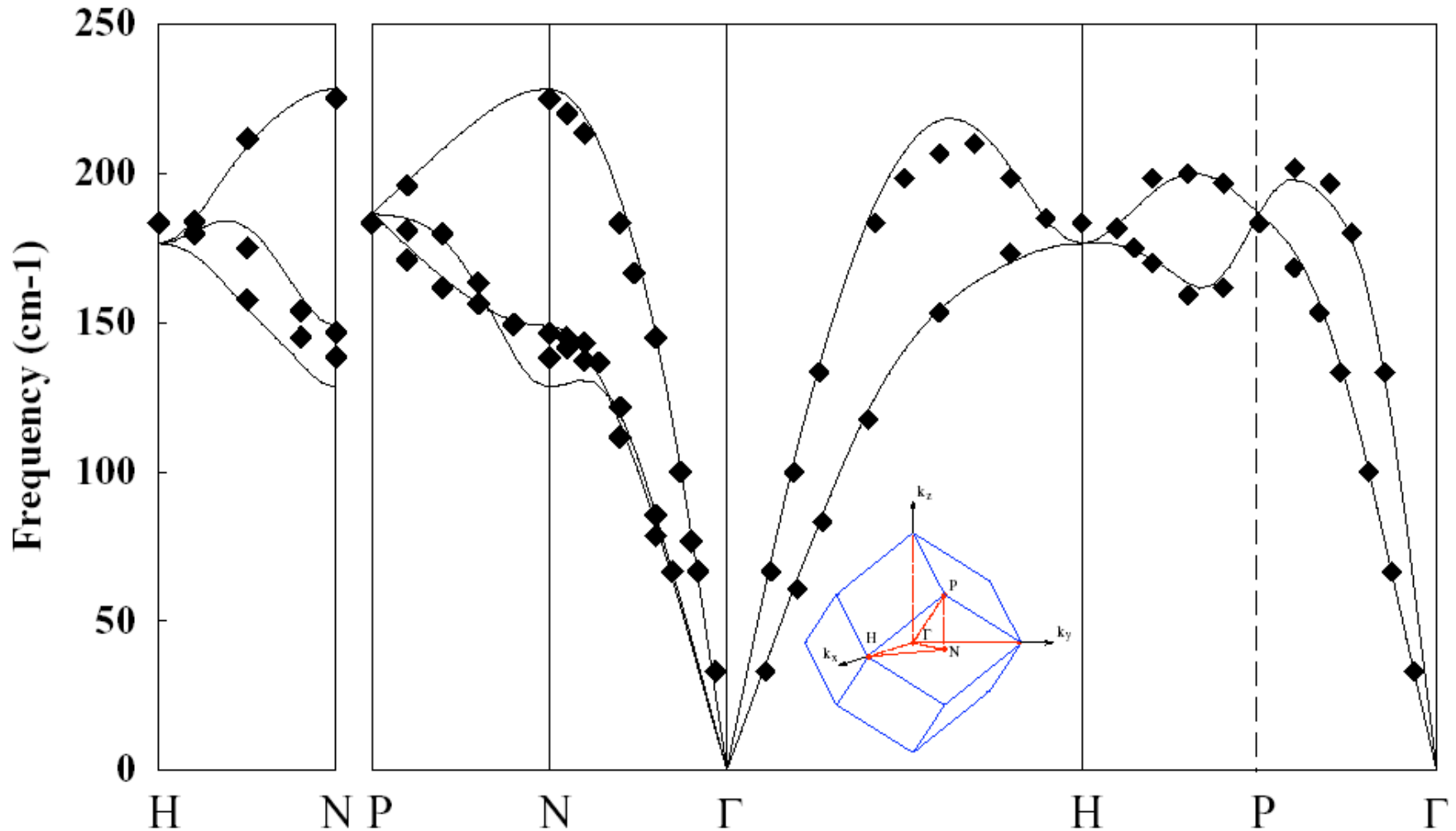
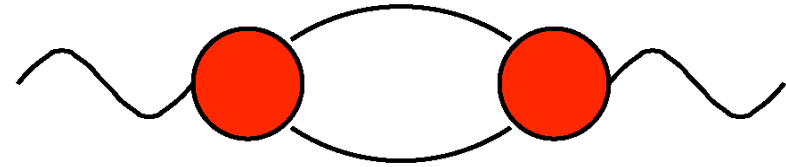
Phonon dispersion in tungsten metal

“first principles” calculation

Density-functional theory

C. Bungaro, thesis, SISSA, 1995

(with Baroni and Gironcoli)



Heat Conduction by Phonons

$$\vec{j}_Q \equiv -\kappa \nabla T$$

Experiments: Eucken

Theories: Einstein, Debye

R. Peierls, thesis, Zurich, 1929

“Phonon gas model”

$$\vec{j}_Q = \sum_k v_k (\hbar \omega_k) N_k(\vec{r})$$

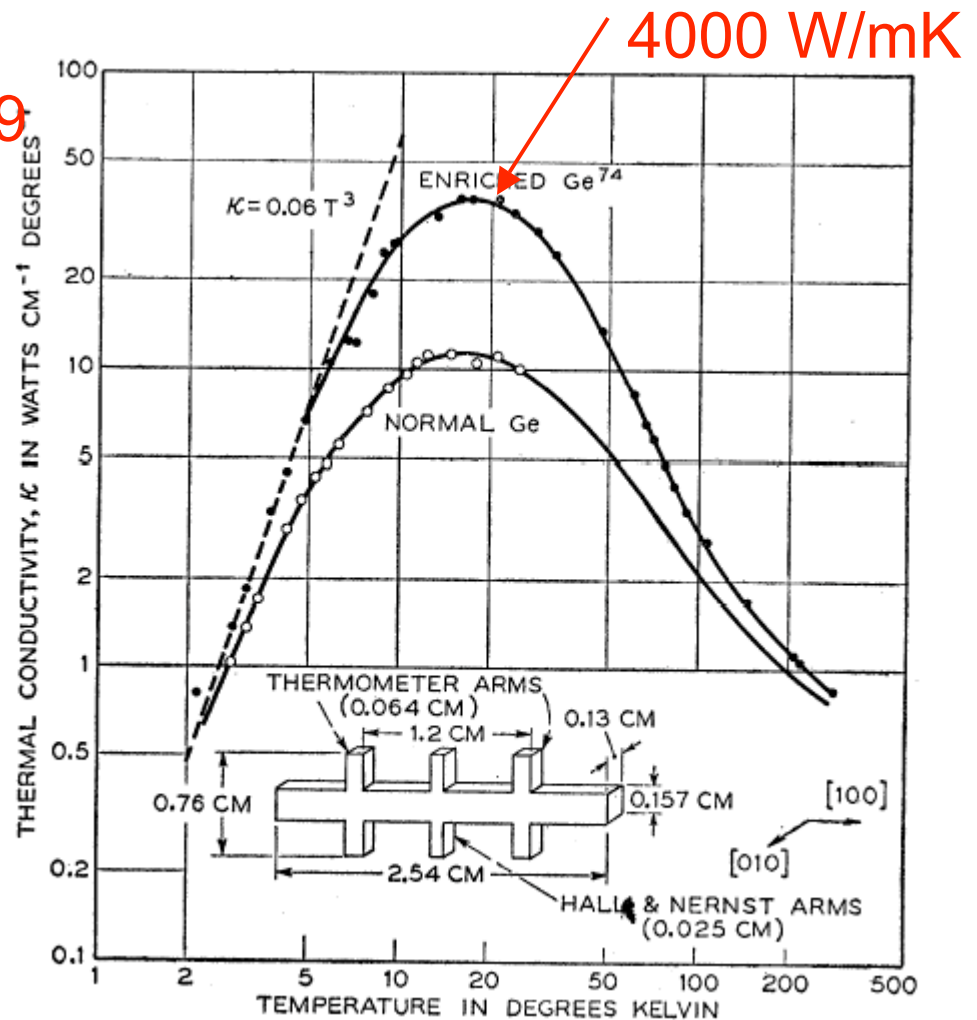
$$N_k(\vec{r}) = n_k [T(\vec{r}) - \vec{v}_k \tau]$$

$$\kappa \approx \frac{1}{3} C(T) \bar{v} \ell(T)$$

Isotopic and Other Types of Thermal Resistance in Germanium*

T. H. GEBALLE AND G. W. HULL

Bell Telephone Laboratories, Murray Hill, New Jersey



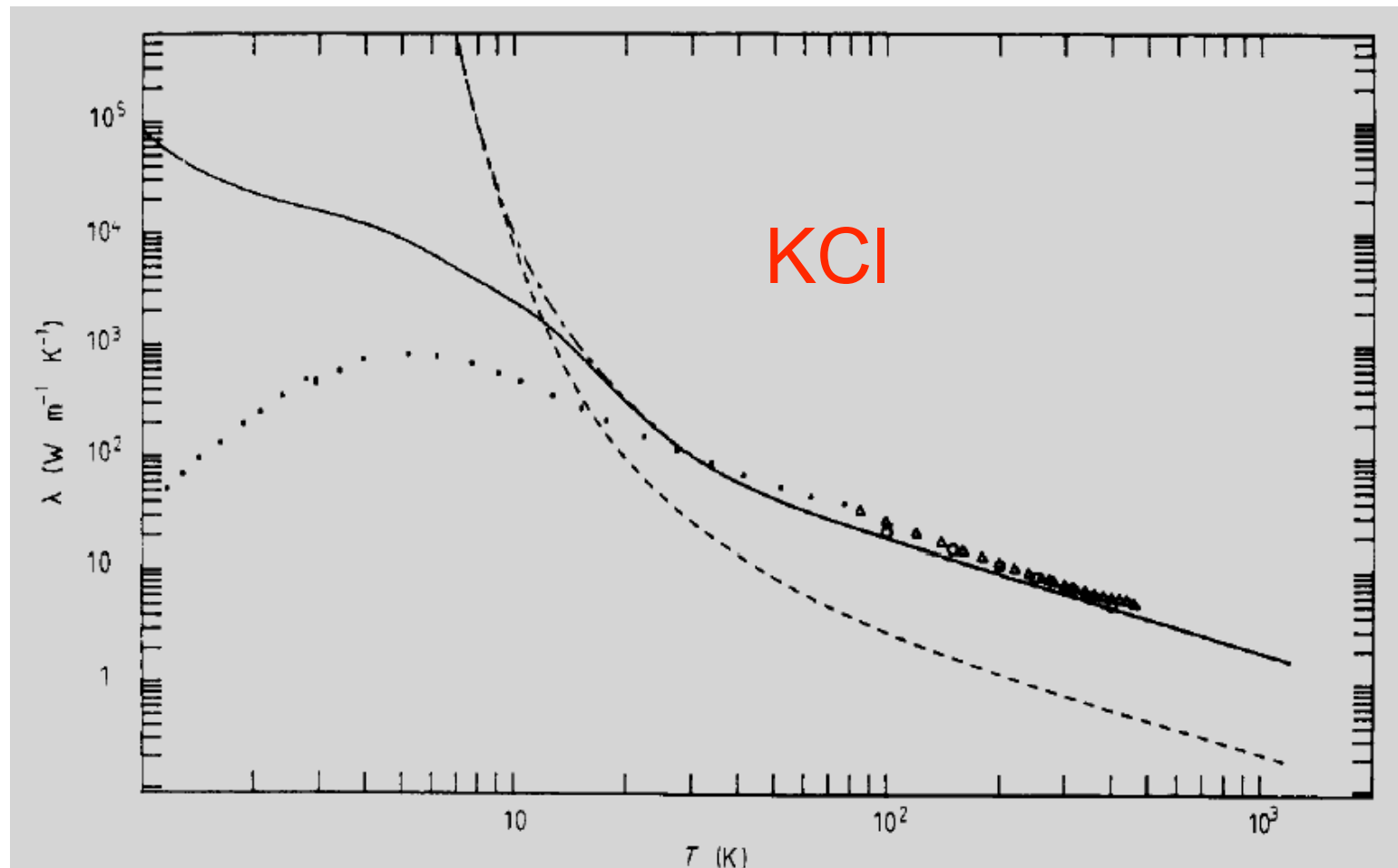
J. Phys. C: Solid State Phys. **20** (1987) 1047–1061.

Calculation of the thermal conductivity of alkali halide crystals

Sune Pettersson

Department of Theoretical Physics, University of Umeå

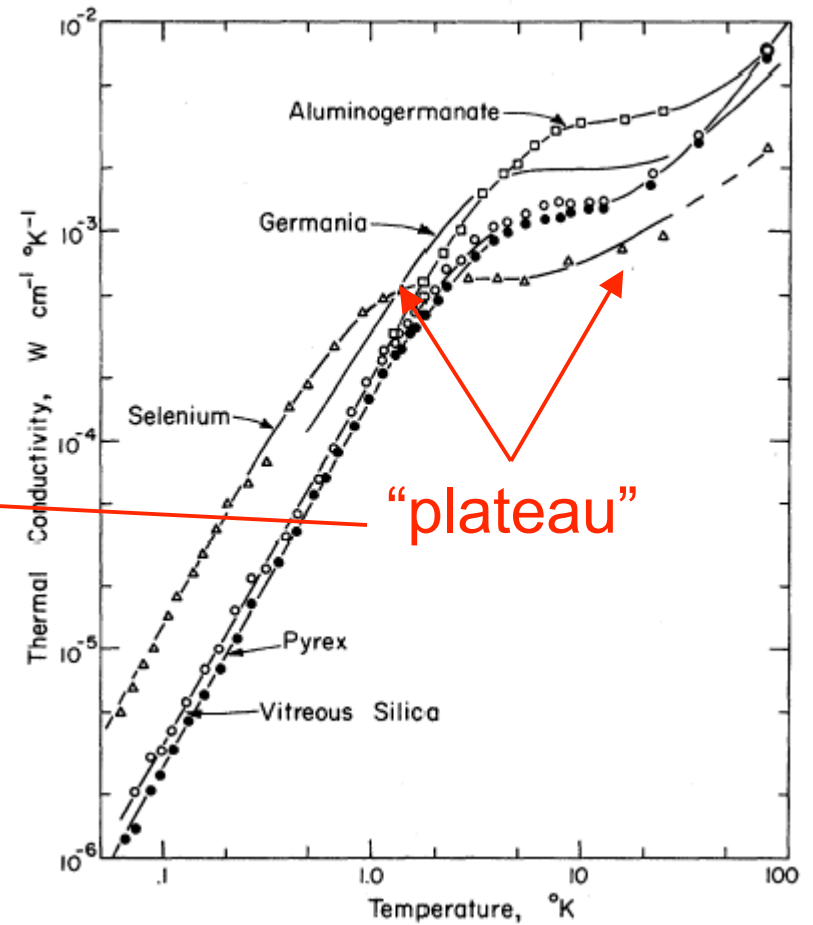
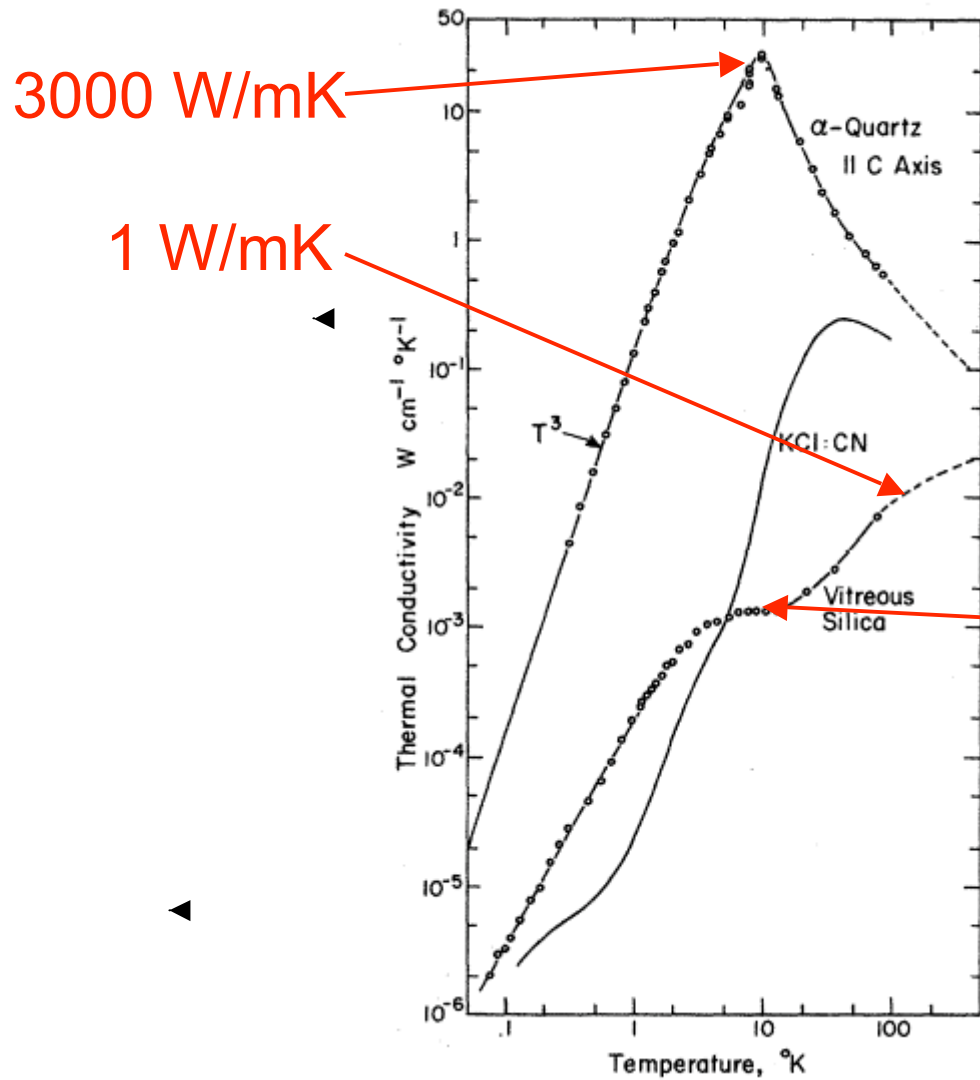
Thesis supervised by Arne Claesson



Thermal Conductivity and Specific Heat of Noncrystalline Solids*

R. C. Zeller† and R. O. Pohl

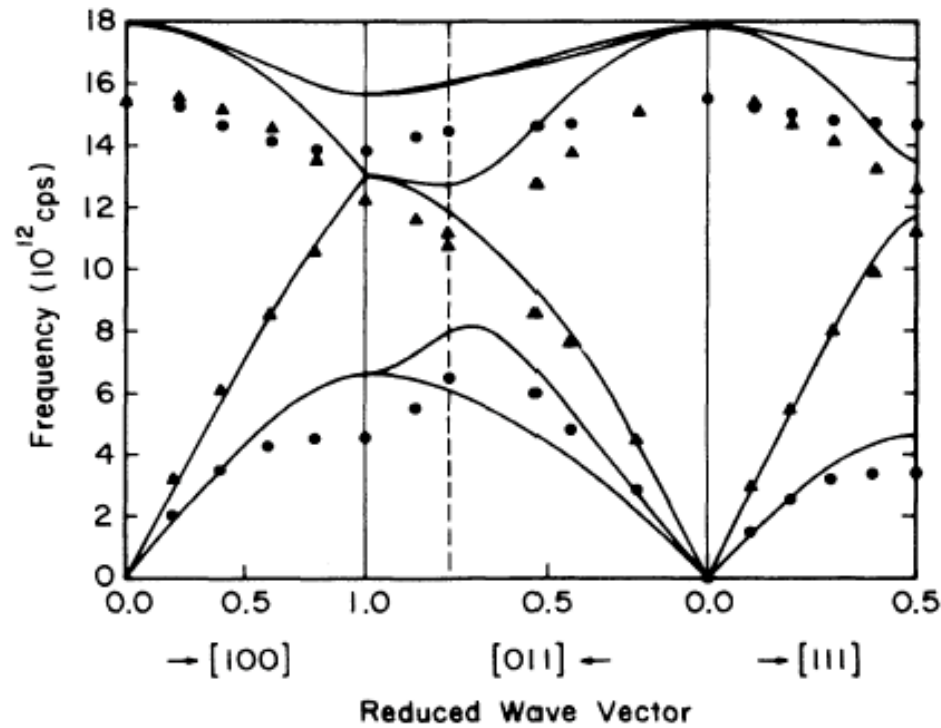
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850



“Phase diagram of silicon by molecular dynamics”

J. Q. Broughton and X. P. Li

Phys. Rev. B 35, 9120-9127 (1987)

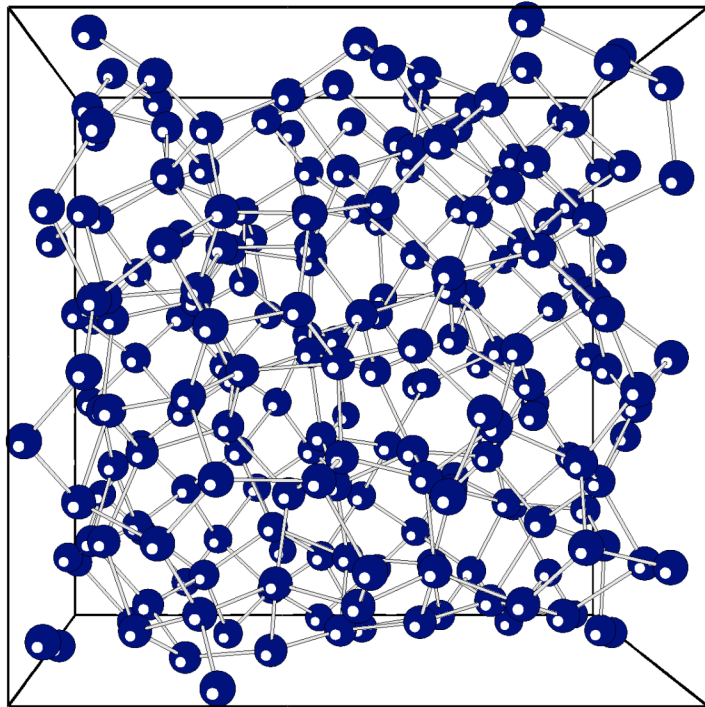


interatomic forces

¹⁶F. H. Stillinger and T. A. Weber, Phys. Rev. B 31, 5262 (1985).

amorphous silicon

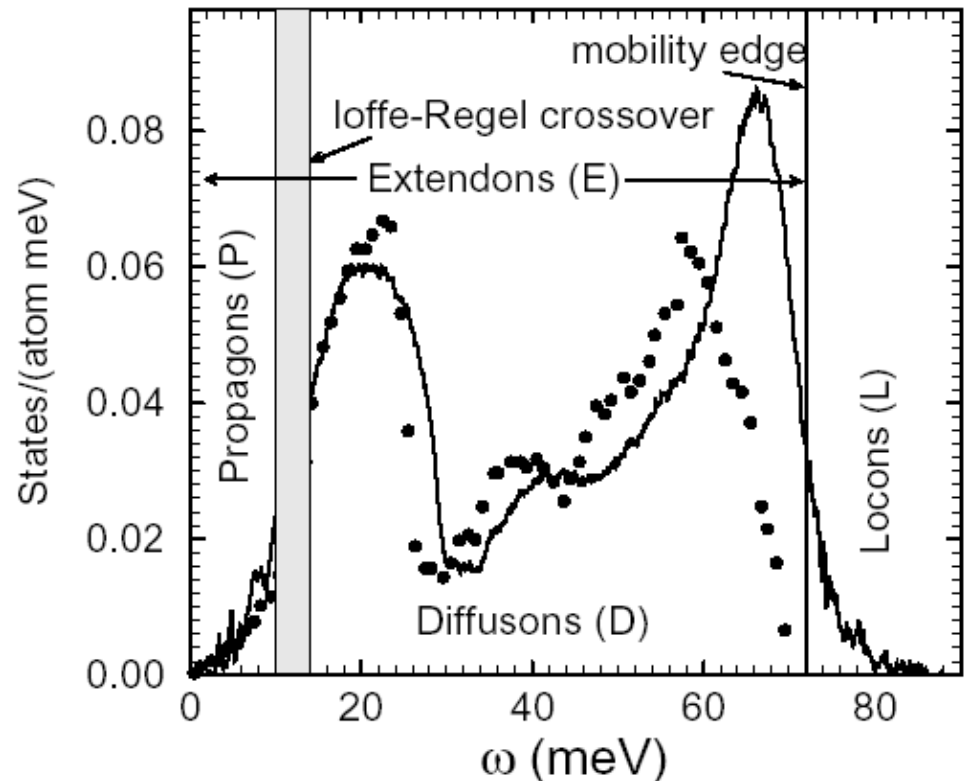
PBA, J. L. Feldman, J. Fabian



atom coordinates

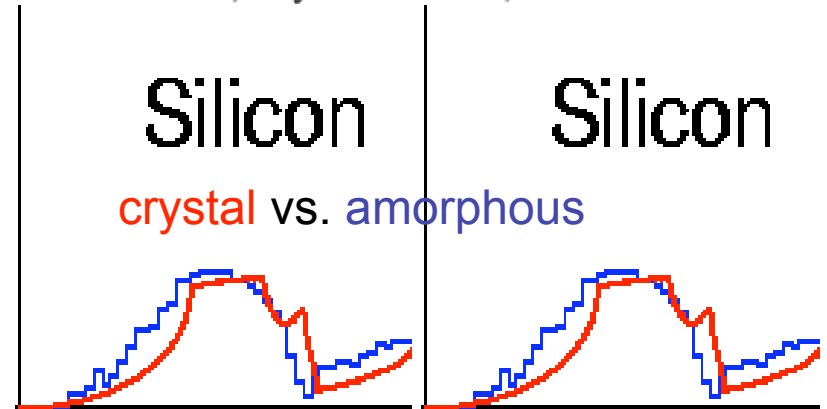
¹⁵F. Wooten, K. Winer, and D. Weaire, Phys. Rev. Lett. **54**, 1392 (1985).

vibrational density of states

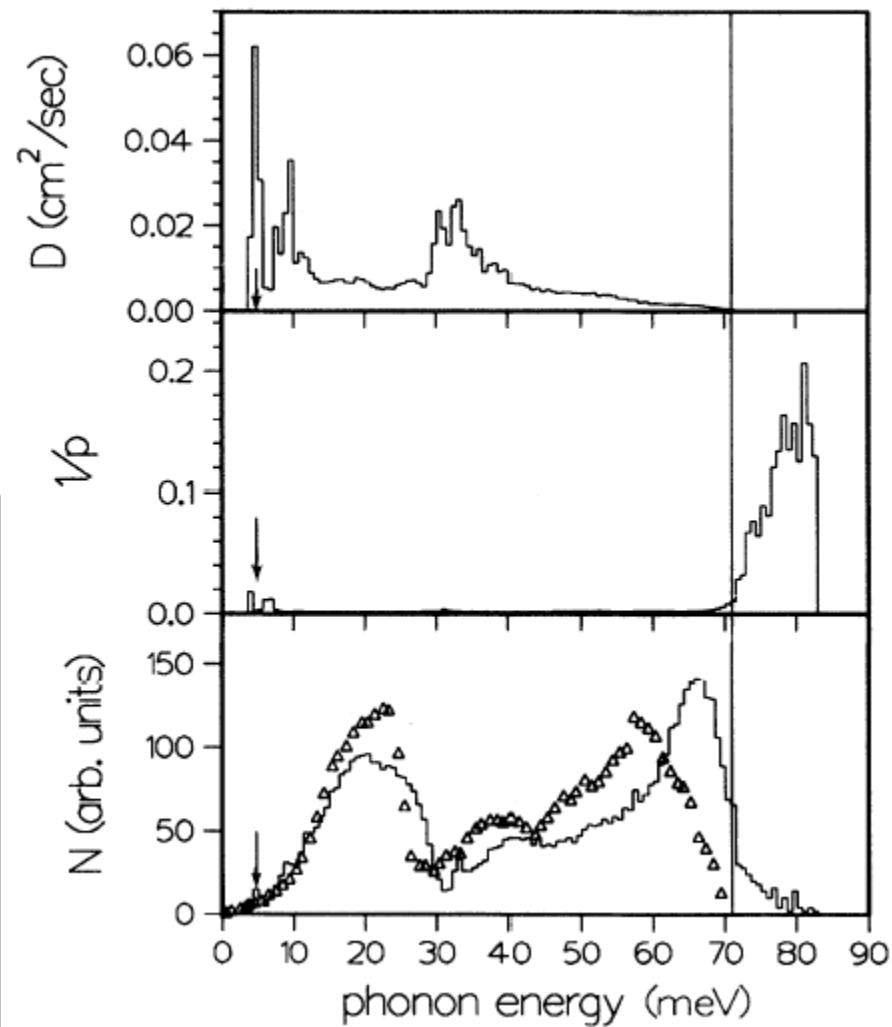
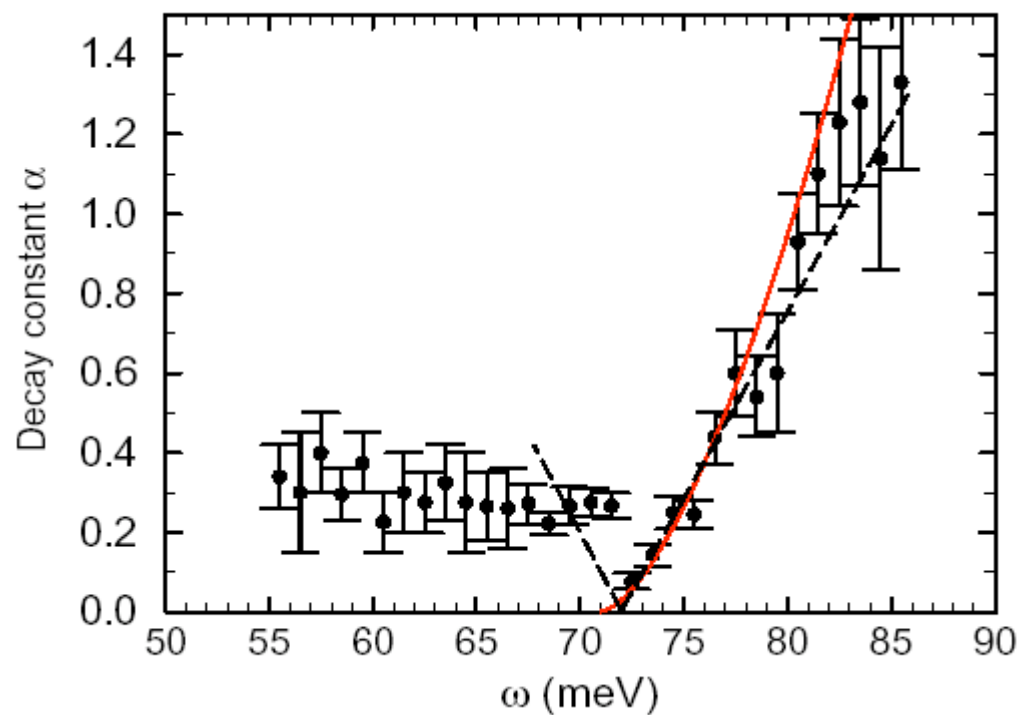


Neutron Scattering data

²⁰W. A. Kamitakahara, C. M. Soukoulis, H. R. Shanks, U. Buchenau, and G. S. Grest, Phys. Rev. B **36**, 6539 (1987).



$$\frac{1}{\rho_i} = \sum_l \left[\sum_{\alpha} \epsilon_i(l, \alpha)^2 \right]^2$$



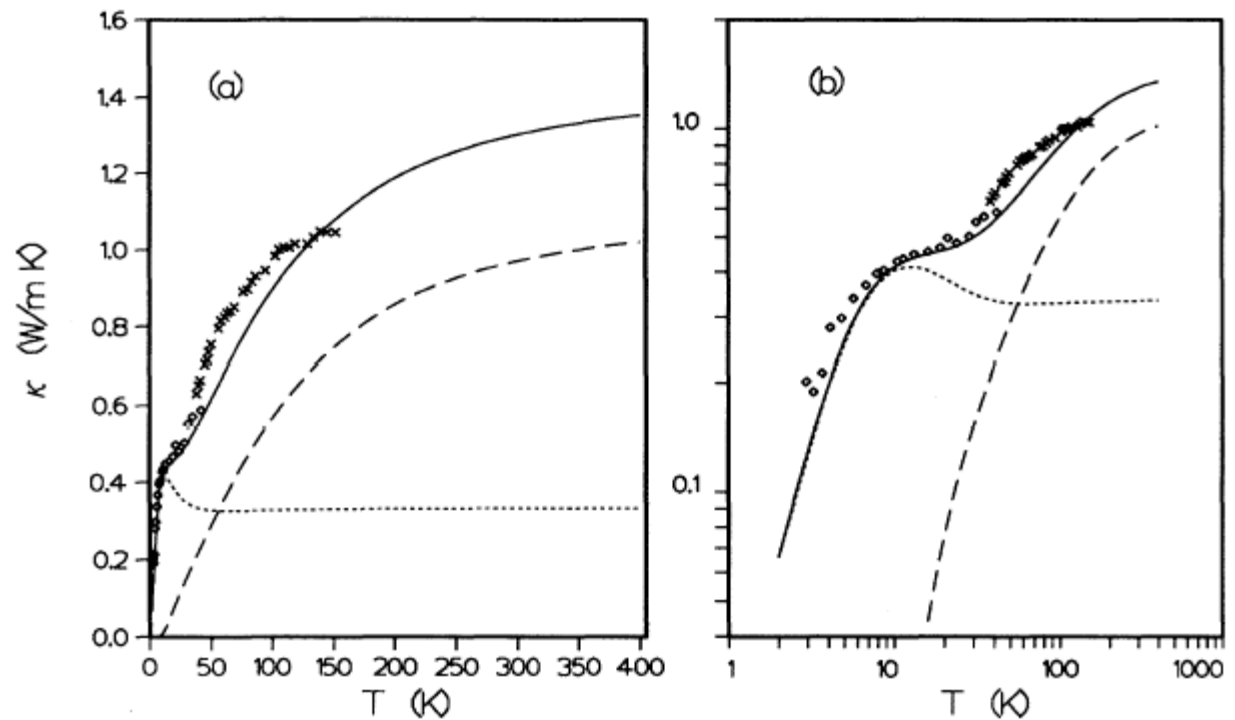
Phonon gas model: $\mathbf{J} = \frac{1}{V} \sum_i \mathbf{v}_i \hbar \omega_i \delta N_i$,

does not apply! Group velocity v_i is undefined!

Lattice analog of Kubo-Greenwood formula (Allen and Feldman, 1987)

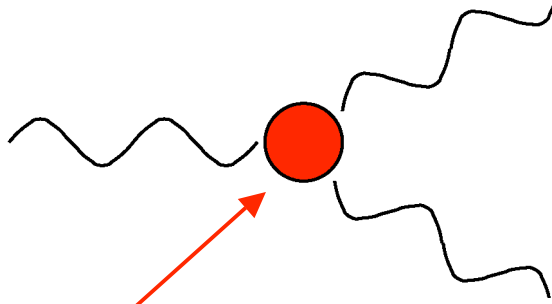
$$\kappa = \frac{1}{V} \sum_i C_i(T) D_i$$

$$D_i = \frac{\pi V^2}{3 \hbar^2 \omega_i^2} \sum_{j \neq i} |S_{ij}|^2 \delta(\omega_i - \omega_j)$$



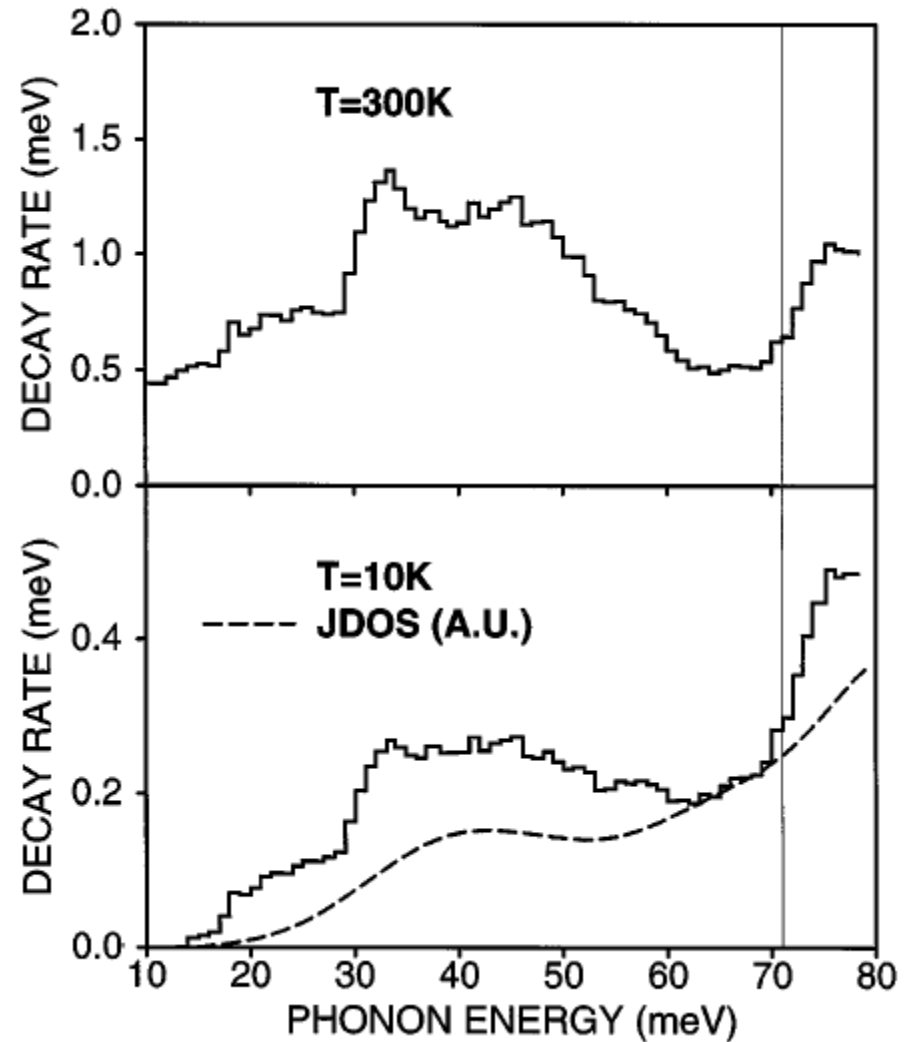
“Anharmonic Decay of Vibrational States in Amorphous Silicon”

Jaroslav Fabian and Philip B. Allen
Phys. Rev. Lett. 77, 3839 (1996)



$$V(j, k, l) = \sum_{a,b,c} \sum_{\alpha\beta\gamma} \frac{\partial^3 V}{\partial u_{a\alpha} \partial u_{b\beta} \partial u_{c\gamma}} \frac{e_{a\alpha}^j}{\sqrt{m_a}} \frac{e_{b\beta}^k}{\sqrt{m_b}} \frac{e_{c\gamma}^l}{\sqrt{m_c}}$$

$$2\Gamma(j) = \frac{\hbar^2 \pi}{4\omega(j)} \sum_{k,l} \frac{|V(j, k, l)|^2}{\omega(k)\omega(l)} \times \left(\frac{1}{2} [1 + n(k) + n(l)] \delta[\omega(j) - \omega(k) - \omega(l)] + [n(k) - n(l)] \delta[\omega(j) + \omega(k) - \omega(l)] \right). \quad (1)$$



Observation of Giant Kohn Anomaly in the One-Dimensional Conductor $K_2Pt(CN)_4Br_{0.3} \cdot 3H_2O$

B. Renker, H. Rietschel, L. Pintschovius, W. Gläser,
P. Brüesch, D. Kuse, and M. J. Rice
Phys. Rev. Lett. 30, 1144 (1973)

Peierls-Overhauser Insulator, or
“band Jahn-Teller system”
partly-filled electron energy band

“Peierls
gap”

