PHYSICS 555 FALL 2003

problem set 3
due Wednesday Oct. 15

1. **Localized impurity state for phonons.** Suppose you have a 1-d chain of atoms, spacing \( a \), mass \( M \), with nearest neighbor springs \( K \). At the origin \( \ell = 0 \) is an impurity atom. The mass is \( M - \Delta M \). The spring constants are unchanged.

   a. Write Newton’s equation of motion for atoms \( \ell = 2, -1, 0, 1, 2 \).

   b. Try whether a solution exists where every atom oscillates at some unknown frequency \( \omega \) and the amplitudes of displacement are \( A \) for atom 0, and decaying with alternating signs in both directions (positive and negative) away from atom 0. In other words, \( u_\ell = (-1)^{|\ell|}\alpha^|\ell| \) where \( \alpha \) is an unknown number less than 1. If such a localized solution exists, find the values of \( A \), \( \alpha \), and \( \omega \).

2. **Green’s function method.** For the same problem, the Green’s function method (presented in class and in Ziman’s book, p. 71ff.) can be used. The equation for the frequency \( \omega \) of oscillation is

\[
\frac{1}{N} \sum_Q \omega^2 - \omega^2_Q = \frac{M}{\Delta M}
\]

where \( \omega_Q \) is the eigenfrequency for a perfect chain with one atom per cell. With only nearest neighbor springs, \( \omega^2_Q \) is \( \omega^2_{\text{max}} \sin^2(Qa/2) \). Ziman’s book has a negative sign on the right – that is because the impurity mass was defined there as \( M + \Delta M \) so \( \Delta M \) has the opposite sign. Show that this gives a solution with the same frequency as in the first problem.

**note:** These results were derived in class by a series of elementary tricks starting from a rigorous but graphical solution of the surface state on the diatomic chain with a light mass termination.